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Editorial overview

Confidence distributions and related themes

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ABSTRACT

This is the guest editors' general introduction to a Special Issue of the Journal of Statistical Planning and Inference, dedicated to confidence distributions and related themes. Confidence distributions (CDs) are distributions for parameters of interest, constructed via a statistical model after analysing the data. As such they serve the same purpose for the frequentist statisticians as the posterior distributions for the Bayesians. There have been several attempts in the literature to put up a clear theory for such confidence distributions, from Fisher's fiducial inference and onwards. There are certain obstacles and difficulties involved in these attempts, both conceptually and operationally, which have contributed to the CDs being slow in entering statistical mainstream. Recently there is a renewed surge of interest in CDs and various related themes, however, reflected in both series of new methodological research, advanced applications to substantive sciences, and dissemination and communication via workshops and conferences. The present special issue of the JSPI is a collection of papers emanating from the *Inference With Confidence* workshop in Oslo, May 2015. Several of the papers appearing here were first presented at that workshop. The present collection includes however also new research papers from other scholars in the field.

The Journal of Statistical Planning and Inference decided in the autumn of 2015 to arrange for a Special Issue on confidence distribution and related themes. After various efforts, by patient authors, referees, and colleagues, along with the customary revision processes, this has resulted in the current collection of eleven journal articles:

1. Cunen et al. (2017a), on CDs and confidence curves for change points, with applications to mediaeval literature and to fisheries sciences;
2. De Blasi and Schweder (2017), on median bias corrections for fine-tuning CDs;
3. Grünwald (2017), on safe probability, leading also to tools for predictions;
4. Hannig et al. (2017), on fusion learning and inter-laboratory analyses;
5. Lewis (2017), on combining inferences, with application to climate statistics;
6. Lindqvist and Taraldsen (2017), on proper uses of improper distributions;
7. Martin (2017), on generalised inference models;
8. Schweder (2017), with an essay on epistemic probability;
9. Shen et al. (2017), on CDs for predictions, in different setups;
10. Taraldsen and Lindqvist (2017), on conditional fiducial models; and
11. Veronese and Melilli (2017), on CDs and their connections to objective Bayes.

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These papers deal with theory and applications for distributional statistical inference, with CDs and fiducial distributions being the central concepts. Quite a few contributions also touch Bayesian angles and connections, however Cunen et al. (2017a), Grünwald (2017), Lewis (2017), Lindqvist and Taraldsen (2017), Taraldsen and Lindqvist (2017) and Veronese and Melilli (2017). In the present general introduction to the Special Issue, by the guest editors, efforts are made both to explain to the broader statistical audience what confidence distributions (CDs) and confidence curves are; why and how they are steadily becoming more popular, in statistical theory and practice; and to briefly place the eleven papers in a broader context. In our article, which is by itself a gentle introduction to the general CD themes, we also attempt to point to aspects and issues and types of application not already contained in the review paper Xie and Singh (2013) and ensuing discussion.

1. The Holy Grail: frequentist posterior distributions

Suppose data are analysed via some model, and that ψ is a parameter of particular interest. Statisticians have many methods in their toolboxes for conducting inference for ψ, such as reaching a point estimate, assessing its precision, setting up tests, along with p-values when of relevance, finding confidence intervals, comparing the ψ with other parameters from other studies, etc. For the frequentist, constructing a distribution for ψ, given the available information, is more problematic, however, also conceptually.

Somehow it appears to be a strict Bayesian privilege to arrive at an appropriate posterior distribution, say $p(\psi \mid \text{data})$—along with the associated difficulties of carrying out Bayesian work in the first place, involving elicitation of prior distributions and combining these with probability distributions of a different kind. Working out a $p(\psi \mid \text{data})$ in the frequentist framework appears to clash with the basic premise that the parameter vector of the model is a fixed but unknown point in the parameter space. This has not stopped scholars from attempting precisely such a feat, called the Holy Grail of parametric statistics by Brad Efron (Efron, 2010). The earliest attempts were by none other than Sir Ronald Fisher, in a series of papers in the 1930ies (Fisher, 1930, 1932, 1933, 1935). Certain obstacles and difficulties were found and pointed to by a number of critical scholars, however, and Fisher did not quite manage to defend his notion of a fiducial distribution for parameters. Indeed the fiducial ideas have been referred to as 'Fisher's biggest blunder'; see Schweder and Hjort (2016, Ch. 6) for an account of the historical development, and also Grünwald (2017, this issue).

There are however other and partly related notions of how to reach proper frequentist posterior distributions, without priors, and the collective labels for a fair portion of these refined and modernised constructions are *confidence distributions* (CDs) and *confidence curves*. There is a clear surge of interest in these methods and in various related themes, regarding both theory and applications. This is witnessed in books and journal articles and by applied advanced work, and is also reflected in high-level workshops and conferences. The *BFF: Bayes, Frequentist, Fiducial* series of conferences (also referred to as 'Best Friends Forever') is reaching a steadily wider audience, with the current list being Shanghai (2014, 2015), Rutgers, New Jersey (2016), Harvard, Massachusetts (2017), Ann Arbor, Michigan (2018), and Duke and SAMSI, North Carolina (2019). There are also special invited sessions at major conferences, etc., dedicated to CDs and BFF themes. Efron (1998) speculates that Fisher's (alleged) biggest blunder might turn into a big hit for the 21st century; see also Efron and Hastie (2016, Ch. 11).

The present special issue of the JSPI is dedicated to such CDs and the growing list of related topics. The collection of papers and the ensuing organisation of the special issue have grown out of one of these conferences, the *Inference With Confidence* workshop in Oslo in May 2015, organised by the research group *FocuStat: Focus Driven Statistical Inference With Complex Data*. Some of the papers appearing in this issue were first presented as invited lectures at this workshop. We have also recruited contributions from other scholars in the field, however, in an attempt to exhibit and see discussed a decent range of the more crucial dimensions of CDs and their increasing scope and usefulness, in methodological and applied statistical work.

"The three revolutions in parametric statistical inference are due to Laplace (1774), Gauss and Laplace (1809–1811) and Fisher (1922)", is the clear opening statement in the two books Hald (1998, 2006). Somewhat boldly, Schweder and Hjort (2016, Preface) claim there is an ongoing fourth revolution in statistics, at the start of the current millennium. This fourth revolution has perhaps a less clear focus than the three drastic methodological changes Hald describes, and is arguably more about the *who* and *what* than about the *how*, but we argue there that CDs and confidence curves have a natural place in the world of statistical computation and communication, also with Big Data. "I wish I'd seen a confidence curve earlier", as tweeted J.M. White, who manages a branch of Facebook's Core Data Science team, in April 2017. We should also make clear that there by necessity are several approaches (partly related and partly competing) to the alleged Holy Grail of reaching posteriors without priors. In addition to the CD theory expounded in Schweder and Hjort (2002, 2003, 2016) and Xie and Singh (2013), with roots all the way back to Fisher in the 1930ies, there is generalised fiducial inference, see Hannig et al. (2016) and Hannig et al. (2017, this issue), along with Lindqvist and Taraldsen (2017, this issue) and Taraldsen and Lindqvist (2017, this issue); as well as the theory of inferential models, cf. Martin and Liu (2015) and Martin (2017, this issue). There is bound to be yet other hybrids and connections, and some of these are touched upon in the present collection of journal articles.

2. What are confidence distributions and confidence curves?

There are several ways in which to motivate, define and construct such CDs, along with associated concepts and functions. Suppose the model for the data y is governed by a parameter vector θ, and that the interest parameter ψ is a function $\psi(\theta)$

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