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Abstract

The axiomatic foundation of probability theory presented by Kolmogorov has been the basis of modern theory for probability and statistics. In certain applications it is, however, necessary or convenient to allow improper (unbounded) distributions, which is often done without a theoretical foundation. The paper reviews a recent theory which includes improper distributions, and which is related to Renyi's theory of conditional probability spaces. It is in particular demonstrated how the theory leads to simple explanations of apparent paradoxes known from the Bayesian literature. Several examples from statistical practice with improper distributions are discussed in light of the given theoretical results, which also include a recent theory of convergence of proper distributions to improper ones.

Keywords: axioms of probability, Bayesian statistics, conditional law, Gibbs sampling, intrinsic Gaussian Markov random fields, marginalization paradox

1. Introduction

Bayes' formula forms the basis of Bayesian statistics. Suppose a parameter θ is of interest, and that we have data x which is supposed to give information about θ. The idea of Bayesian inference is to first express one's prior knowledge (some would call it *uncertainty*) of θ in the form of a *prior distribution*, commonly given in the form of a density function $\pi(\theta)$, and then combine this knowledge with the new knowledge provided by the data x. The

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