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Conditional fiducial models

G. Taraldsen *, B.H. Lindqvist

Department of Mathematical Sciences, Norwegian University of Science and Technology, 7046 Trondheim, Norway

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ABSTRACT

The fiducial is not unique in general, but we prove that in a restricted class of models it is uniquely determined by the sampling distribution of the data. It depends in particular not on the choice of a data generating model. The arguments lead to a generalization of the classical formula found by Fisher (1930). The restricted class includes cases with discrete distributions, the case of the shape parameter in the Gamma distribution, and also the case of the correlation coefficient in a bivariate Gaussian model. One of the examples can also be used in a pedagogical context to demonstrate possible difficulties with likelihood-, Bayesian-, and bootstrap-inference. Examples that demonstrate non-uniqueness are also presented. It is explained that they can be seen as cases with restrictions on the parameter space. Motivated by this the concept of a conditional fiducial model is introduced. This class of models includes the common case of *iid* samples from a one-parameter model investigated by Hannig (2013), the structural group models investigated by Fraser (1968), and also certain models discussed by Fisher (1973) in his final writing on the subject.
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1. Introduction

Fisher (1930, p. 528–531) introduced the term fiducial distribution of a parameter by first criticizing the convention of using a non-informative Bayesian prior. He explains that a constant prior is just as arbitrary as any other prior by consideration of a constant prior for a reparametrized model.

There has been considerable progress since 1930 on the theory related to the choice of a non-informative prior based on symmetry considerations (Jeffreys, 1946), entropy (Jaynes, 2003), or other information theoretic arguments (Bernardo, 1979; Berger et al., 2009). The argument by Fisher (1930) remains, however, perfectly valid also today: There is information in any particular choice for the prior.

As an alternative, for cases where there is lack of prior information, Fisher introduced his fiducial argument. According to Fisher (1950, p. 428): *The importance of the paper lies, however, in setting forth a new mode of reasoning from observations to their hypothetical causes.* Today, this new mode of reasoning is still in development and different lines of arguments have been published (Schweder and Hjort, 2002; Taraldsen and Lindqvist, 2013; Xie and Singh, 2013; Martin and Liu, 2014; Hannig et al., 2016).

The original argument (Fisher, 1930, p. 532) starts by consideration of the relation

$$U = F(X \mid \theta) \tag{1}$$

where X is the maximum likelihood estimator of the parameter θ, and $F(x \mid \theta) = P^{\theta}(X \leq x)$. From this Fisher (1930, p. 534) shows, given certain additional assumptions, that the fiducial probability density $c(\theta)$ of θ is given by differentiation of F

* Corresponding author.
E-mail address: Gunnar.Taraldsen@ntnu.no (G. Taraldsen).

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with respect to θ:

$$c(\theta \mid x) = -\partial_\theta F(x \mid \theta) \tag{2}$$

The assumptions ensure in particular that $U \sim Uniform(0, 1)$ so discrete distributions are ruled out.

It is common to use the term *posterior* instead of the more cumbersome *Bayesian posterior distribution* when the meaning is clear from the context. We will likewise use the term *fiducial* instead of the more cumbersome *fiducial distribution* when the meaning is clear from the context.

In this paper, we generalize formula (2) to include cases with discrete distributions, and to include cases where the initial equation (1) is replaced by an alternative equation which allows more general U. This leads to a class of models where the fiducial is unique and determined by the sampling distribution of the data as stated in Theorem 1. The generalization in this class of models is consistent with more general definitions of a fiducial model as considered by Taraldsen and Lindqvist (2013) and Hannig et al. (2016).

Motivated by cases where the conclusion in Theorem 1 fails we introduce in Section 5 the general concept of a conditional fiducial model which is a generalization of the concept of a fiducial model (Taraldsen and Lindqvist, 2013; Hannig et al., 2016): It is a fiducial model with an added condition. This is also motivated directly by examples presented originally by Fisher (1973, p. 138) and also later by Seidenfeld (1992). Conceptually, this can be seen as the most important result in this paper.

It is shown, by general examples, that this gives a possible approach for fiducial inference in multivariate observations and for group models. In the group case a natural condition is given by a maximal invariant, and the inference from the conditional fiducial model then coincides with the Bayes posterior from the right Haar prior. For the conditional fiducial models considered the resulting fiducial is again unique, but only when both the fiducial model and the condition are given. The fiducial is, however, not uniquely given by the sampling distribution of the data.

2. Fiducial inference

Fiducial theory, as presented here, is closely connected to methods for simulation and testing of statistical inference procedures. This is by itself good motivation for the current interest in fiducial theory. Computer simulation of models has become an integral part of modern statistical practice, and fiducial arguments are particularly well adapted to this practice.

A fiducial model (U, χ), as defined in this paper, is defined by a fiducial equation

$$x = \chi(u, \theta) \tag{3}$$

for an *observation* x together with a probability distribution P_U^θ for u given the *parameter* θ. The definition given by Eq. (3), and its relation to other possible definitions, is discussed in more detail by Taraldsen and Lindqvist (2016, 2015, 2013). The relation (3) replaces the corresponding relation $u = F(x, \theta)$ in Eq. (1) used originally by Fisher (1930). The distribution P_U^θ is the Monte Carlo law of the fiducial model. Sampling from the statistical model $\{P_X^\theta\}$ is defined by $x = \chi(u, \theta)$ for a simulated sample u from P_U^θ and a fixed model parameter θ. A fiducial model is hence exactly what is needed for simulating data from a statistical model: It is a data generating model.

Sampling from the fiducial is defined by solving the fiducial equation to obtain $\theta = \hat{\theta}(u, x)$ for each simulated sample u and the given observed x. Existence and uniqueness of the solution is assumed, and the fiducial model is then said to be simple. For a *simple fiducial model* it is also assumed that the Monte Carlo law P_U^θ does not depend on θ. More general cases can be considered, but the theory then splits into several alternative theories (Dempster, 1968; Shafer, 2008; Fraser, 1968; Wilkinson, 1977; Dawid and Stone, 1982; Taraldsen and Lindqvist, 2013; Martin and Liu, 2014; Hannig et al., 2016).

Formally, in the case of a simple fiducial model, the fiducial distribution can be defined as follows. Let $U^x \sim \mathrm{P}_U^\theta$ and define the fiducial random quantity $\Theta^x = \hat{\theta}(U^x, x)$. The law of Θ^x is the fiducial distribution or simply the fiducial. The interpretation is as for the posterior in a Bayesian analysis: It is an epistemic probability law derived for the parameter based on the observation. The role of the prior and the statistical model in Bayesian analysis is replaced by the use of the fiducial model (3) in fiducial inference. The fiducial is obtained in this case without a prior distribution for the parameter.

A further advantage of the fiducial approach in the case of a simple fiducial model is that independent samples are produced directly from independent sampling from P_U^θ. Bayesian simulations most often come as dependent samples from a Markov chain. Fraser (1968) presents many non-trivial models which can be used to exemplify this for applied concrete problems.

The fiducial argument can also be used within a frequentist frame of inference. This is similar to how the Bayesian machine is used to produce frequentist methods in cases which would otherwise be intractable. The fiducial approach has then the advantage that it comes equipped with a method for simulating data from the statistical model. This is exactly what is needed for repeated testing of any given inference procedure suggested for the model: Fiducial, Bayesian, or obtained by other arguments. This is a most convenient circumstance.

It is clear, and this will be exemplified in the next section, that there exist many different fiducial models for a given statistical model. This is related to the fact that there exist many different algorithms for the simulation of data from a given statistical model.

The concept of a fiducial model as used in this paper is hence not so that it is uniquely determined by the likelihood function. Fisher (1973) insisted that the fiducial distribution should be defined in terms of the likelihood function. An

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