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Varying coefficient partially nonlinear models with nonstationary regressors

Zhiyong Zhou [a,*], Zhengyan Lin [b]

[a] Department of Mathematics and Mathematical Statistics, UmeåUniversity, Umeå, Sweden
[b] School of Mathematical Sciences, Zhejiang University, Hangzhou, China

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ABSTRACT

We study a varying coefficient partially nonlinear model in which the regressors are generated by the multivariate unit root processes. A profile nonlinear least squares estimation procedure is applied to estimate the parameter vector and the functional coefficients. Under some mild conditions, the asymptotic distribution theory for the resulting estimators is established. The rate of convergence for the parameter vector estimator depends on the properties of the nonlinear regression function. However, the rate of convergence for the functional coefficients estimator is the same and enjoys the super-consistency property. Furthermore, a simulation study is conducted to investigate the finite sample performance of the proposed estimating procedures.

1. Introduction

In the past two decades, various nonparametric and semiparametric estimation techniques have been applied to model stationary time series (see Fan and Gijbels, 1996; Härdle et al., 2000; Fan and Yao, 2003; Gao, 2007; Li and Racine, 2007; and the references therein). However, as pointed out in the literature, the nonstationarity is a very important empirical feature in many economic and financial time series. For example, both prices and exchange rates are nonstationary. Thus, in recent years, there has been much interest in nonparametric and semiparametric models with nonstationary covariates, existing literature include Phillips and Park (1998), Karlsen and Tjøstheim (2001), Juhl and Xiao (2005), Karlsen et al. (2007), Cai et al. (2009), Xiao (2009), Wang and Phillips (2009a, b, 2016), Sun and Li (2011), Chen et al. (2012), Gao and Phillips (2013), Sun et al. (2013), Chen et al. (2015), Gao et al. (2015), Liang et al. (2015), Dong et al. (2016) and Li et al. (2017).

In this paper, we tackle a general class of semiparametric models with nonstationary covariates. Specifically, we focus on a varying coefficient partially nonlinear model with the form

$$Y_t = \alpha_t^T \beta(Z_t) + g(X_t, \gamma) + u_t, \qquad (1)$$

where α_t is a d-dimensional I(1) vector, $\beta(\cdot)$ is a d-dimensional vector of unspecified smooth functions, Z_t is a scalar stationary variable, $g : \mathbb{R} \times \mathbb{R}^m \to \mathbb{R}$ is a known nonlinear function, X_t is a scalar I(1) variable, γ is a $m \times 1$ vector of constant parameters that lies in the parameter set Γ and u_t is a stationary error term. Here, the notation I(1) refers to the integrated of order one time series or the unit root process. The advantage of the semiparametric model compared with the nonparametric one, is that it attenuates the "curse of dimensionality" problem. In addition, Model (1) that we discuss provides a very flexible framework in nonstationary time series, and it covers various linear and nonlinear time series models with nonstationarity.

* Corresponding author.
E-mail address: zhiyong.zhou@umu.se (Z. Zhou).

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For example, when $g(X_t, \gamma) = \gamma X_t$, (1) reduces to semi-varying coefficient models with nonstationary regressors which has been investigated by Gu and Liang (2014) and Li et al. (2017). When $g(X_t, \gamma) = 0$, (1) becomes the functional coefficient models with nonstationarity, which has been studied by Cai et al. (2009), Xiao (2009), Sun and Li (2011), Gao and Phillips (2013), Sun et al. (2013) and Chen et al. (2015). When the varying coefficient part equals 0, the model reduces to the nonlinear parameter regressions model with integrated time series which has been systematically investigated in existing literature such as Park and Phillips (1999, 2001), Chang and Park (2011) and Chan and Wang (2015).

The main focus of this paper is to investigate the semiparametric estimation for both the parameter vector γ and the functional coefficients $\beta(\cdot)$, and then derive the associated asymptotic distribution theory. When the covariates in the model (1) are i.i.d, the profile nonlinear least squares estimation procedure was proposed by Li and Nie (2008) and Li and Mei (2013). In that case, the resulting parametric vector estimator is consistent and asymptotic normal with rate of convergence $\sqrt{n}$. Meanwhile, the nonparametric estimator for functional coefficients enjoys the asymptotically normality with usual rate of convergence $\sqrt{nh}$. In the present paper, we extend the results to the nonstationary time series case. The extension involves the asymptotic theory for the time series with nonstationarity. Moreover, as the model contains a nonlinear parameter part, similar to Park and Phillips (1999, 2001), we show that the rate of convergence for the parameter vector estimator depends on the properties of the known nonlinear regression function $g(\cdot, \cdot)$. However, the specification of $g(\cdot, \cdot)$ given in our paper does not affect the asymptotic property of the nonparametric estimator for the functional coefficients. We show that the rate of convergence for the nonparametric estimator is $n\sqrt{h}$, which is common in the nonstationary time series case. Our results complement existing literature concerning nonlinear nonstationary time series (see, for example, Park and Phillips, 2001; Juhl and Xiao, 2005; Cai et al., 2009; Xiao, 2009; Wang and Phillips, 2009a, b; Sun and Li, 2011; Chen et al., 2012; Sun et al., 2013 and Li et al., 2017).

The paper is organized as follows. Section 2 introduces the profile nonlinear least squares estimation method. The asymptotic distribution theory for the proposed estimators are given in Section 3. A simulation study is presented in Section 4. Section 5 is devoted to the conclusion. Finally, the tedious definitions, the technical lemmas and the detailed proofs are postponed to the Appendix.

2. Estimation method

We use the profile nonlinear least squares estimation method proposed in Li and Nie (2008) to estimate the parameter vector γ and the functional coefficients $\beta(\cdot)$. Throughout the paper, we let γ_0 and $\beta_0(\cdot)$ be the true parameter vector and functional coefficients.

Firstly, we fix γ, then

$$Y_t - g(X_t, \gamma) = \alpha_t^T \beta(Z_t) + u_t. \tag{2}$$

We adopt the local linear fitting method (Fan and Gijbels, 1996) to estimate the functional coefficient $\beta_0(\cdot)$. Assuming that $\beta_0(\cdot)$ has second order derivative, we have the following Taylor expansion:

$$\beta_0(z) \approx \beta_0(z_0) + \beta_0^{'}(z_0)(z - z_0),$$

for z in a small neighborhood of z_0. The local linear estimate of $\Psi(z_0) = (\beta_0(z_0)^T, h\beta_0^{'}(z_0)^T)^T$ for given γ is defined by minimizing the weighted loss function (with respect to Ψ):

$$L_n(\Psi|\gamma) = \sum_{t=1}^{n} \left\{Y_t - g(X_t, \gamma) - V_t(z_0)^T \Psi\right\}^2 K_h(Z_t - z_0),$$

where Ψ is a $2d$-dimensional vector, $V_t(z_0) = \begin{pmatrix} \alpha_t \\ h^{-1}(Z_t - z_0)\alpha_t \end{pmatrix}$, and $K_h(\cdot) = K(\cdot/h)/h$ with $K(\cdot)$ being the kernel function and h being the bandwidth.

Let $Y = (Y_1, \ldots, Y_n)^T$, $g(\gamma) = (g(X_1, \gamma), \ldots, g(X_n, \gamma))^T$, $M_0 = (\alpha_1^T \beta_0(Z_1), \ldots, \alpha_n^T \beta_0(Z_n))^T$, $W(z_0) = \mathrm{diag}(K_h(Z_1 - z_0), \ldots, K_h(Z_n - z_0))$, and

$$\alpha(z_0) = \begin{pmatrix} V_1(z_0)^T \\ \vdots \\ V_n(z_0)^T \end{pmatrix} = \begin{pmatrix} \alpha_1^T & h^{-1}(Z_1 - z_0)\alpha_1^T \\ \vdots & \vdots \\ \alpha_n^T & h^{-1}(Z_n - z_0)\alpha_n^T \end{pmatrix}.$$

Then, the local linear estimate of $\Psi(z_0) = (\beta_0(z_0)^T, h\beta_0^{'}(z_0)^T)^T$ with given γ can be expressed as

$$\hat{\Psi}(z_0, \gamma) = \left[\alpha(z_0)^T W(z_0) \alpha(z_0)\right]^{-1} \alpha(z_0)^T W(z_0)(Y - g(\gamma)). \tag{3}$$

As a consequence, the estimate of the functional coefficients $\beta_0(z_0)$ is

$$\hat{\beta}(z_0, \gamma) = (\mathrm{I}_d, 0_{d\times d})\hat{\Psi}(z_0, \gamma) = (\mathrm{I}_d, 0_{d\times d})\left[\alpha(z_0)^T W(z_0)\alpha(z_0)\right]^{-1} \alpha(z_0)^T W(z_0)(Y - g(\gamma)), \tag{4}$$