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Performance analysis of decentralized multi-channel feedback systems for active noise control in free space

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ABSTRACT

This paper investigates the performance of decentralized multi-channel feedback analog control systems, which are flexible and economic for practical applications of active noise control. The generalized Nyquist theorem and Gerschgorin circle theorem are used to derive a sufficient stability condition in terms of the predefined maximum noise amplification and the geometrical configuration of the independent controllers, and the noise reduction performance of the multi-channel system is predicted with the design and geometrical configuration of the independent controllers. Simulation and experimental results are presented to illustrate the effectiveness of the proposed analysis.

1. Introduction

In many practical situations, a multi-channel active noise control (ANC) system with many secondary sources and error sensors is needed to achieve global control [1,2]. A common way to implement a multi-channel control system is to use the centralized control strategy, which uses signals from all error sensors and the secondary path transfer functions between all secondary sources and all error sensors to adjust the output of each secondary source [3]. When the number of secondary sources and error sensors becomes large, the complexity of centralized controllers makes the implementation of such a control strategy difficult or expensive. To reduce the complexity, the decentralized strategy can be used, where each independent single channel controller is designed based on its own corresponding secondary path transfer function and error sensor [4]. The design of the independent controller does not take into account the acoustic coupling from other secondary sources, so it has benefits of simple deign and flexible hardware. However, there might be a performance loss and instability problems induced by neglecting the interactions caused by the acoustic coupling.

A general theoretical analysis of decentralized feedforward controllers has been developed by Elliott et al. and a conservative condition for the stability of the overall system using the Gerschgorin circle theorem has been derived [3]. For adaptive feedback active control systems, practical stability conditions, which take into account the geometrical configuration of the secondary sources and error sources, have been derived from the small gain theorem and the Nyquist criterion by Leboucher et al. [5]. For the non-adaptive decentralized feedback control with independent design, Crosdidier and Morari proposed the μ-measure to predict the stability of the system and to measure the performance degradation caused by the interaction [6]. However, there is a lack of such analysis for non-adaptive decentralized multi-channel feedback ANC systems, in particular, in term of the noise amplification caused by the waterbed effect [7,8].

Being compared to digital controllers, analog controllers have advantages of smaller phase lag and low cost in implementations for large systems. The background of the current research is to apply the decentralized multi-channel analog feedback system to electrical transformer noise control, where each analog feedback controller is designed individually in advance to have predefined noise reduction and noise amplification caused by the waterbed effect, and these independent single channel analog feedback controllers are then configured geometrically to constitute a decentralized multi-channel feedback system to achieve global control. The stability condition, the noise attenuation and amplification properties of the non-adaptive decentralized system in terms of the geometrical configuration of the independent controllers and the noise reduction properties of the independent controllers need to be explored for successful practical applications of the system, which are the objectives of the paper.

Global attenuation of the sound power of an active noise control system depends on the noise attenuation performance at error sensors and the arrangement (number and locations) of secondary

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sources and error sensors [9]. Many papers have been published on the optimization methods for the arrangement of secondary sources and error sensors [10,11]. Similar as Ref. [5], this paper will not discuss the physical system optimization of the whole control system which guarantees minimization at error sensors resulting in a global control, but rather focus on the noise attenuation at the error sensors and the influence by the acoustic coupling on them for given geometrical configurations of the independent controllers, which is also an important part of the active control system for transformer noise control.

In the remainder of this paper, the overall sensitivity transfer function of the decentralized analog narrowband feedback control system is analyzed first, then practical stability conditions, which are in terms of the maximum noise amplification and the geometrical configuration of the independent controllers, are derived using the generalized Nyquist theorem and Gerschgorin circle theorem. The noise reduction performance of the multi-channel system is predicted based on the independent design and geometrical configuration of the independent controllers. Finally, simulations and experiments with four secondary sources and four error sensors are described, simulations with nine secondary sources and nine error sensors are given, and practical effectiveness of the proposed analysis is discussed.

2. Theoretical analysis

A block diagram of the decentralized multi-channel feedback control system is shown in Fig. 1, where $\mathbf{H}(s)$ denotes the transfer functions of N single channel analog controllers diagonally as,

$$\mathbf{H}(s)=\begin{bmatrix} H_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & H_N \end{bmatrix}. \tag{1}$$

$\mathbf{G}(s)$ denotes the secondary path transfer functions, including the transfer functions of the power amplifiers, the physical transfer impedances between secondary sources and error sensors, and the transfer functions of the conditioning hardware for the error sensors,

$$\mathbf{G}(\mathbf{s})=\begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1N} \\ \vdots & \ddots & & \vdots \\ G_{N1} & G_{N2} & \cdots & G_{NN} \end{bmatrix}, \tag{2}$$

where element G_{ij} represents the transfer function between the controller i and error sensor j. $\mathbf{D}(s)$ denotes the primary noise signal vector, and is written as

$$\mathbf{D}(s)=[d_1,d_2,\cdots,d_N]^T. \tag{3}$$

$\mathbf{E}(s)$ denotes the error signal vector,

$$\mathbf{E}(s)=[e_1,e_2,\cdots,e_N]^T, \tag{4}$$

and $\mathbf{Y}(s)$ denotes the output signal vector generated by the secondary sources,

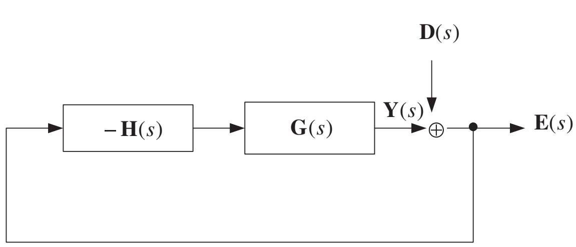

Fig. 1. Block diagram of the decentralized multi-channel feedback control system.

$$\mathbf{Y}(s)=[y_1,y_2,\cdots,y_N]^T. \tag{5}$$

Let $s = j\omega$, the sensitivity function $S_i(j\omega)$ of the ith independent single channel controller can be written as [12]

$$S_i(j\omega)=1/[1+H_i(j\omega)G_{ii}(j\omega)], \tag{6}$$

and the corresponding noise reduction is given by [12]

$$NR_i(j\omega)=20\log_{10}|1/S_i(j\omega)|. \tag{7}$$

Introduce a diagonal matrix

$$\mathbf{G_I}(j\omega)=\begin{bmatrix} G_{11}(j\omega) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & G_{NN}(j\omega) \end{bmatrix}, \tag{8}$$

where the diagonal element is the secondary path frequency response of each independent single channel controller, and

$$\mathbf{S_I}(j\omega)=\begin{bmatrix} S_1(j\omega) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & S_N(j\omega) \end{bmatrix}=[\mathbf{I}+\mathbf{G_I}(j\omega)\mathbf{H}(j\omega)]^{-1}, \tag{9}$$

where the diagonal element is the sensitivity function of the independent single channel controller, then the diagonal matrix $\mathbf{H}(j\omega)$ can be expressed as

$$\mathbf{H}(j\omega)=[\mathbf{S_I}^{-1}(j\omega)-\mathbf{I}]\mathbf{G_I}^{-1}(j\omega). \tag{10}$$

With decentralized control, the error signals $\mathbf{E}(j\omega)$ in Fig. 1 can be written as

$$\begin{aligned}\mathbf{E}(j\omega)&=[\mathbf{I}+\mathbf{G}(j\omega)\mathbf{H}(j\omega)]^{-1}\mathbf{D}(j\omega)\\ &=[\mathbf{I}+\mathbf{G}(j\omega)(\mathbf{S_I}^{-1}(j\omega)-\mathbf{I})\mathbf{G_I}^{-1}(j\omega)]^{-1}\mathbf{D}(j\omega).\end{aligned} \tag{11}$$

The matrix $[\mathbf{I}+\mathbf{G}(j\omega)(\mathbf{S_I}^{-1}(j\omega)-\mathbf{I})\mathbf{G_I}^{-1}(j\omega)]^{-1}$ is called the overall sensitivity function matrix with all loops closed [4], and can be expressed as the following equation by using Eqs. (2) and (8),

$$\mathbf{S}^c(j\omega)=\begin{bmatrix} \frac{1}{S_1(j\omega)} & \left(\frac{1}{S_2(j\omega)}-1\right)\frac{G_{12}(j\omega)}{G_{22}(j\omega)} & \cdots & \left(\frac{1}{S_N(j\omega)}-1\right)\frac{G_{1N}(j\omega)}{G_{NN}(j\omega)} \\ \left(\frac{1}{S_1(j\omega)}-1\right)\frac{G_{21}(j\omega)}{G_{11}(j\omega)} & \frac{1}{S_2(j\omega)} & \cdots & \left(\frac{1}{S_N(j\omega)}-1\right)\frac{G_{2N}(j\omega)}{G_{NN}(j\omega)} \\ \vdots & \cdots & \cdots & \vdots \\ \left(\frac{1}{S_1(j\omega)}-1\right)\frac{G_{N1}(j\omega)}{G_{11}(j\omega)} & \left(\frac{1}{S_2(j\omega)}-1\right)\frac{G_{N2}(j\omega)}{G_{22}(j\omega)} & \cdots & \frac{1}{S_N(j\omega)} \end{bmatrix}^{-1} \tag{12}$$

In this equation, the element $S^c_{ij}(j\omega)$ represents the sensitivity transfer function between the error signal e_i and the primary noise d_j. Eq. (12) indicates that the performance of the decentralized multi-channel feedback control system can be expressed in terms of the sensitivity function of each independent controller and the secondary path transfer functions.

To illustrate the above derivations, take the simplest decentralized system with two independent controllers as an example, that is $N = 2$. From Eq. (12), the sensitivity function $S^c_{11}(j\omega)$ between the error signal e_1 and the primary noise d_1 can be calculated by

$$S^c_{11}(j\omega)=1/[1+H_1(j\omega)G_{11}(j\omega)(1-\gamma(j\omega)C_2(j\omega))], \tag{13}$$

where

$$\begin{aligned}&\gamma(j\omega)=G_{12}(j\omega)G_{21}(j\omega)G_{11}^{-1}(j\omega)G_{22}^{-1}(j\omega),\\ &C_2(j\omega)=H_2(j\omega)G_{22}(j\omega)/[1+H_2(j\omega)G_{22}(j\omega)].\end{aligned} \tag{14}$$

The sensitivity function $S^c_{12}(j\omega)$ between the error signal e_1 and the primary noise d_2 can be calculated similarly as

$$\begin{aligned}S^c_{12}(j\omega)=&-G_{12}(j\omega)G_{22}^{-1}(j\omega)C_2(j\omega)/[1+H_1(j\omega)G_{11}(j\omega)(1\\ &-\gamma(j\omega)C_2(j\omega))].\end{aligned} \tag{15}$$