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Nonparametric circular quantile regression

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Abstract

We discuss nonparametric estimation of conditional quantiles of a circular distribution when the conditioning variable is either linear or circular. Two different approaches are pursued: inversion of a conditional distribution function estimator, and minimization of a smoothed check function. Local constant and local linear versions of both estimators are discussed.

Simulation experiments and a real data case study are used to illustrate the usefulness of the methods.

Keywords: Check Function, Circular Quantile, Circular Distribution Function, Predictive Interval, Wind Turbine
2000 MSC: 62G07 - MSC 62G08

1. Introduction

Quantile regression focuses on estimating either the conditional median or other quantiles of the response variable. When compared with ordinary least squares regression, we can say that quantile estimates are: a) more robust when the distributions of the covariates and/or error terms are heavy tailed; b) more easily interpretable if the conditional distributions are asymmetric. However, quantile regression is typically used to gain insights on the whole underlying conditional distribution through the estimation of various percentiles. Similarly, predictive intervals are often determined by estimating pairs of extreme conditional quantiles.

Methods used for quantile regression range from fully parametric — in the simplest case, we have a normal distributed response, along with linear quantile curves — to fully nonparametric, where local smoothing of observed quantiles is carried out, see, for example, Jones and Hall (1990). Usually, either the quantile curves or response distribution is estimated nonparametrically. Jones and Noufaily (2013) provide a critical account of the literature.

In practice, as it happens for the majority of circular statistics indices, a set of quantiles can be estimated for each possible choice of the origin. In some situations there may be external information, or data could be confined to a small arc of the circle (reproducing an euclidean-like scenario), but more generally (as in our example) we could choose the origin according to a *minimum width* criterion. That is, for any specified quantile, the origin is chosen so as to minimise the width corresponding to the estimated interval. In any case, we observe that a change of the origin simply generates a linear shift on the cumulative distribution function (CDF) values, i.e. in the quantile order. Such an equivariance form links estimates based on same data but with different origin.

Also, the definition, and then the estimation, of the circular CDF is not an obvious extension of the standard theory. It is perhaps for these reasons that quantile regression seems unexplored in the circular setting, even from a parametric perspective. The CDF of a random angle Θ having density f is defined as $F(\theta) := \int_{-\pi}^{\theta} f(u)du$, $\theta \in [-\pi, \pi)$; this implies that $F(-\pi) = 0$ and $F(\pi) = 1$. When f is seen as a periodic function having $\mathbb{R}$ as its support, then

$$\lim_{a \to \pm\infty} F(a) = \pm\infty,$$

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