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Numerical simulation of parametric sound generation and its application to length-limited sound beam

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ABSTRACT

In this study we propose a simulation model for predicting the nonlinear sound propagation of ultrasound beams over a distance of a few hundred wavelengths, and we estimate the beam profile of a parametric array. Using the finite-difference time-domain method based on the Yee algorithm with operator splitting, axisymmetric nonlinear propagation was simulated on the basis of equations for a compressible viscous fluid. The simulation of harmonic generation agreed with the solutions of the Khokhlov–Zabolotskaya–Kuznetsov equation around the sound axis except near the sound source. As an application of the model, we estimated the profiles of length-limited parametric sound beams, which are generated by a pair of parametric sound sources with controlled amplitudes and phases. The simulation indicated a sound beam with a narrow truncated array length and a width of about one-quarter to half that of regular a parametric beam. This result confirms that the control of sound source conditions changes the shape of a parametric beam and can be used to form a torch like low-frequency sound beam.

1. Introduction

The radiation and propagation of two intense ultrasound beams at different but neighboring frequencies generate not only higher harmonic sounds but also sum and difference frequency sounds by the nonlinearity of a medium. A parametric array is an application of this phenomenon and forms a sharp directive sound beam with the difference frequency within the ultrasonic beams [1,2]. There have been a number of fundamental and application studies on parametric arrays [3–10]. In particular, a length-limited parametric sound beam [8] is of great interest owing to its narrow truncated array length. Although such a beam has been experimentally demonstrated, it has not been discussed in detail.

For nonlinear sound beams including parametric beams, a number of analytical and numerical approaches have been developed [11–21]. The Khokhlov–Zabolotskaya–Kuznetsov (KZK) equation [11] is frequently used to describe the nonlinear propagation of ultrasound beams. This equation can only predict one-way propagation of sound in a field relatively distant from a sound source and near the beam axis due to a the parabolic approximation of nonlinear sound wave equation.

A time-domain sound simulation is a powerful tool for probing the transient behavior of sound propagation in a time domain. In particular, the finite-difference time-domain method based on the Yee algorithm (Yee-FDTD) [22] is the most commonly used method for investigating linear sound propagation. For nonlinear sound propagation, some modified models have been proposed, for example, a model with the bulk modulus depending on the local particle velocity [12]. However this model is not suitable for obtaining the bulk modulus as a scalar value from the particle velocity which is a vector value. In addition, the simulation gives a distorted pulse shape caused by numerical dispersion.

An FDTD method based on the Westervelt equation has been used to simulate nonlinear propagation [13]. The model is simple because it uses only one wave equation, whereas the general Yee-FDTD method uses two equations of pressure and velocity. However more memory is required for the computation because the model uses five previous time-step values. A nonlinear full-wave simulation model with an FDTD method based on the conservation form of hydrodynamic equations has been applied to focused sound pulse propagation [14]. The simulation did not make use of the advantages of the full-wave equation, because sound pressure was predicted by a second-order the approximation of the equation of state from the density.

Numerical fluid dynamical approaches, such as the TVD method with high resolution and the MacCormack method, have been applied to not only strong nonlinear sound propagation but also the analysis of other nonlinear sound phenomena, for example, acoustic streaming and acoustic radiation pressure [15–17]. These methods, however, require large computational power.

Recently, there have been some new numerical approaches to nonlinear sound simulations [18–20]. The constrained interpolation

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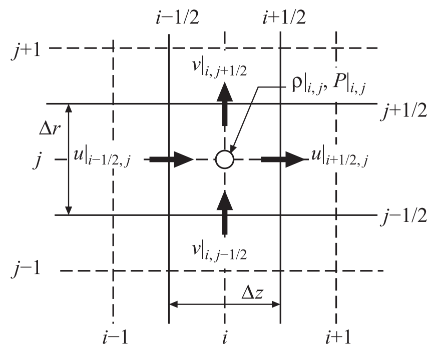

Fig. 1. Staggered grid system. Particle velocities u and v are defined at the cell boundaries ($i \pm 1/2,j$) and ($i,j \pm 1/2$), respectively, and density ρ and pressure P are defined at the cell center (i,j).

profile (CIP) method [18,19], which use field values as well as their spatial gradients, has been introduced in acoustics. The CIP simulation indicates less numerical dispersion even for small number of cells, however the simulation has high computational complexity and exhibits numerical energy dissipation at high frequencies. A Monte Carlo simulation, which is one of the particle-based methods, has been used to describe nonlinear propagation in a mean free path [20]. However, large-scale simulations require a large computational time.

As stated above, it is difficult to accurately simulate nonlinear propagation over a long distance, such as that for the field of a parametric array. In this study, we attempt to simulate nonlinear sound propagation over a distance of a few hundred wavelengths based on fluid dynamics using the Yee-FDTD method with operator splitting and to predict the sound fields of the parametric array. In addition, as an application of the proposed simulation method, we simulate the generation of a length-limited parametric sound beam and estimate its profile.

2. Model equations and simulation method

2.1. Governing equations

We first consider the nonlinear propagation of sound radiated from an axisymmetric circular sound source at $z = 0$. The sound propagation is described as compressible viscous fluid motion and an ideal gas. The particle velocity vector $\mathbf{u}$ with axial and radial components u and v, density ρ and pressure P consisting of static and perturbation components are expressed in axisymmetric cylindrical coordinates (z,r) and in the time domain t as follows:

$$\frac{\partial\tilde{u}}{\partial\tilde{t}}+\tilde{u}\frac{\partial\tilde{u}}{\partial\tilde{z}}+\tilde{v}\frac{\partial\tilde{u}}{\partial\tilde{r}}=-\frac{1}{\tilde{\rho}}\left\{\frac{1}{\gamma}\frac{\partial\widetilde{P}}{\partial\tilde{z}}-\frac{1}{Re}\left(\frac{\partial\tilde{\tau}_{zz}}{\partial\tilde{z}}+\frac{\partial\tilde{\tau}_{zr}}{\partial\tilde{r}}+\frac{\tilde{\tau}_{zr}}{\tilde{r}}\right)\right\}, \tag{1}$$

$$\frac{\partial\tilde{v}}{\partial\tilde{t}}+\tilde{u}\frac{\partial\tilde{v}}{\partial\tilde{z}}+\tilde{v}\frac{\partial\tilde{v}}{\partial\tilde{r}}=-\frac{1}{\tilde{\rho}}\left\{\frac{1}{\gamma}\frac{\partial\widetilde{P}}{\partial\tilde{r}}-\frac{1}{Re}\left(\frac{\partial\tilde{\tau}_{rz}}{\partial\tilde{z}}+\frac{\partial\tilde{\tau}_{rr}}{\partial\tilde{r}}+\frac{\tilde{\tau}_{rr}-\tilde{\tau}_{\theta\theta}}{\tilde{r}}\right)\right\}, \tag{2}$$

$$\frac{\partial\tilde{\rho}}{\partial\tilde{t}}+\tilde{u}\frac{\partial\tilde{\rho}}{\partial\tilde{z}}+\tilde{v}\frac{\partial\tilde{\rho}}{\partial\tilde{r}}=-\tilde{\rho}\left(\frac{\partial\tilde{u}}{\partial\tilde{z}}+\frac{\partial\tilde{v}}{\partial\tilde{r}}+\frac{\tilde{v}}{\tilde{r}}\right), \tag{3}$$

$$\frac{\partial\widetilde{P}}{\partial\tilde{t}}+\tilde{u}\frac{\partial\widetilde{P}}{\partial\tilde{z}}+\tilde{v}\frac{\partial\widetilde{P}}{\partial\tilde{r}}=-\gamma\left\{\widetilde{P}\left(\frac{\partial\tilde{u}}{\partial\tilde{z}}+\frac{\partial\tilde{v}}{\partial\tilde{r}}+\frac{\tilde{v}}{\tilde{r}}\right)-\frac{1}{Re}(\gamma-1)\widetilde{Q}\right\}, \tag{4}$$

with normalized variables with the notation $\tilde{\cdot}$, $\tilde{t}=\omega_0 t$, $\tilde{z}=k_0 z$, $\tilde{r}=k_0 r$, $\tilde{\rho}=\rho/\rho_0$, $\tilde{\mathbf{u}}=\mathbf{u}/c_0$, $\widetilde{P}=P/P_0$, $\tilde{\boldsymbol{\tau}}=\boldsymbol{\tau}/(\mu_0\omega_0)$, $\widetilde{T}=T/T_0$, $\tilde{\mu}=\mu/\mu_0$ and $\tilde{\kappa}=\kappa/\kappa_0$, where T is the absolute temperature, ω_0 is the characteristic angular frequency of the sound, $k_0 = \omega_0/c_0$ is the characteristic wave number, γ is the specific heat ratio, C_p is the specific heat coefficient at a constant pressure and c is the sound speed. The subscript 0 in the variables designates the quantities at atmospheric pressure (1 atm) and at temperature $T = T_0 = 298.15$ K (=25 °C). The components of the stress tensor $\boldsymbol{\tau}$, the term Q related to viscous dissipation and thermal conductivity, and the equation of state are written as

$$\tilde{\tau}_{zz}=-\frac{2}{3}\tilde{\mu}\left(\frac{\partial\tilde{u}}{\partial\tilde{z}}+\frac{\partial\tilde{v}}{\partial\tilde{r}}+\frac{\tilde{v}}{\tilde{r}}\right)+2\tilde{\mu}\frac{\partial\tilde{u}}{\partial\tilde{z}}, \tag{5}$$

$$\tilde{\tau}_{rr}=-\frac{2}{3}\tilde{\mu}\left(\frac{\partial\tilde{u}}{\partial\tilde{z}}+\frac{\partial\tilde{v}}{\partial\tilde{r}}+\frac{\tilde{v}}{\tilde{r}}\right)+2\tilde{\mu}\frac{\partial\tilde{v}}{\partial\tilde{r}}, \tag{6}$$

$$\tilde{\tau}_{\theta\theta}=-\frac{2}{3}\tilde{\mu}\left(\frac{\partial\tilde{u}}{\partial\tilde{z}}+\frac{\partial\tilde{v}}{\partial\tilde{r}}+\frac{\tilde{v}}{\tilde{r}}\right)+2\tilde{\mu}\frac{\tilde{v}}{\tilde{r}}, \tag{7}$$

$$\tilde{\tau}_{zr}=\tilde{\tau}_{rz}=\tilde{\mu}\left(\frac{\partial\tilde{u}}{\partial\tilde{r}}+\frac{\partial\tilde{v}}{\partial\tilde{z}}\right), \tag{8}$$

$$\widetilde{Q}=\tilde{\tau}_{zz}\frac{\partial\tilde{u}}{\partial\tilde{z}}+\tilde{\tau}_{zr}\frac{\partial\tilde{u}}{\partial\tilde{r}}+\tilde{\tau}_{zr}\frac{\partial\tilde{v}}{\partial\tilde{z}}+\tilde{\tau}_{rr}\frac{\partial\tilde{v}}{\partial\tilde{r}}+\frac{\tilde{\kappa}}{(\gamma-1)Pr}\left(\frac{\partial^2\widetilde{T}}{\partial\tilde{z}^2}+\frac{\partial^2\widetilde{T}}{\partial\tilde{r}^2}+\frac{1}{\tilde{r}}\frac{\partial\widetilde{T}}{\partial\tilde{r}}\right), \tag{9}$$

$$\widetilde{P}=\tilde{\rho}\widetilde{T}, \tag{10}$$

where the Reynolds and Prandtl numbers are defined as $Re = \rho_0 c_0/(\mu_0 k_0)$ and $Pr = \mu_0 C_p/\kappa_0$, respectively, μ is the shear viscosity and κ is the thermal conductivity. In the present model, it is assumed that the medium is isotropic and that μ, κ, C_p and γ are independent of location and temperature.

The above equations are summarized in the following compact form:

$$\frac{\partial\widetilde{\mathbf{E}}_1}{\partial\tilde{t}}=\widetilde{\mathbf{F}}_{A1}+\widetilde{\mathbf{F}}_{AD1}+\widetilde{\mathbf{F}}_{D1}, \tag{11}$$

$$\frac{\partial\widetilde{\mathbf{E}}_2}{\partial\tilde{t}}=\widetilde{\mathbf{F}}_{A2}+\widetilde{\mathbf{F}}_{AD2}+\widetilde{\mathbf{F}}_{D2}, \tag{12}$$

where

$$\widetilde{\mathbf{E}}_1=[\tilde{u},\quad \tilde{v}]^{\mathrm{T}}, \tag{13}$$

$$\widetilde{\mathbf{E}}_2=\left[\tilde{\rho},\quad \widetilde{P}\right]^{\mathrm{T}}, \tag{14}$$

$$\widetilde{\mathbf{F}}_{A1}=-\frac{1}{\gamma\tilde{\rho}}\begin{bmatrix}\frac{\partial\widetilde{P}}{\partial\tilde{z}}\\ \frac{\partial\widetilde{P}}{\partial\tilde{r}}\end{bmatrix}, \tag{15}$$

$$\widetilde{\mathbf{F}}_{A2}=-\begin{bmatrix}\tilde{\rho}\left(\frac{\partial\tilde{u}}{\partial\tilde{z}}+\frac{\partial\tilde{v}}{\partial\tilde{r}}+\frac{\tilde{v}}{\tilde{r}}\right)\\ \gamma\widetilde{P}\left(\frac{\partial\tilde{u}}{\partial\tilde{z}}+\frac{\partial\tilde{v}}{\partial\tilde{r}}+\frac{\tilde{v}}{\tilde{r}}\right)\end{bmatrix}, \tag{16}$$

$$\widetilde{\mathbf{F}}_{AD1}=-\begin{bmatrix}\tilde{u}\frac{\partial\tilde{u}}{\partial\tilde{z}}+\tilde{v}\frac{\partial\tilde{u}}{\partial\tilde{r}}\\ \tilde{u}\frac{\partial\tilde{v}}{\partial\tilde{z}}+\tilde{v}\frac{\partial\tilde{v}}{\partial\tilde{r}}\end{bmatrix}, \tag{17}$$

$$\widetilde{\mathbf{F}}_{AD2}=-\begin{bmatrix}\tilde{u}\frac{\partial\tilde{\rho}}{\partial\tilde{z}}+\tilde{v}\frac{\partial\tilde{\rho}}{\partial\tilde{r}}\\ \tilde{u}\frac{\partial\widetilde{P}}{\partial\tilde{z}}+\tilde{v}\frac{\partial\widetilde{P}}{\partial\tilde{r}}\end{bmatrix}, \tag{18}$$

$$\widetilde{\mathbf{F}}_{D1}=\frac{1}{\tilde{\rho}Re}\begin{bmatrix}\frac{\partial\tilde{\tau}_{zz}}{\partial\tilde{z}}+\frac{\partial\tilde{\tau}_{zr}}{\partial\tilde{r}}+\frac{\tilde{\tau}_{zr}}{\tilde{r}}\\ \frac{\partial\tilde{\tau}_{zr}}{\partial\tilde{z}}+\frac{\partial\tilde{\tau}_{rr}}{\partial\tilde{r}}+\frac{\tilde{\tau}_{rr}-\tilde{\tau}_{\theta\theta}}{\tilde{r}}\end{bmatrix}, \tag{19}$$

$$\widetilde{\mathbf{F}}_{D2}=\frac{1}{Re}\begin{bmatrix}0\\ \gamma(\gamma-1)\widetilde{Q}\end{bmatrix}, \tag{20}$$