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An alternative discrete skew Laplace distribution

Alessandro Barbiero *

Department of Economics, Management and Quantitative Methods, Università degli Studi di Milano, via Conservatorio, 7 - 20122 Milan, Italy

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ABSTRACT

In this paper, an alternative discrete skew Laplace distribution is proposed, which is derived by using the general approach of discretizing a continuous distribution while retaining its survival function. The distribution's properties are explored and it is compared to a Laplace distribution on integers recently proposed in the literature. The issues related to the sample estimation of its parameters are discussed, with a particular focus on the maximum likelihood method and large-sample confidence intervals based on Fisher's information matrix; a modified version of the method of moments is presented along with the method of proportion, which is particularly suitable for such a discrete model. Two hypothesis tests are suggested. A Monte Carlo simulation study is carried out to assess the statistical properties of these inferential techniques. Applications of the proposed model to real data are given as well.

1. Introduction

Researchers in many fields regularly encounter variables that are discrete in nature or in practice. In life testing experiments, for example, it is sometimes impossible or inconvenient to measure the life length of a device on a continuous scale; in many practical situations, in fact, reliability data are measured in terms of the number of runs, cycles, or shocks the device sustains before failing. In survival analysis, one may record the number of days of survival for patients since therapy, or the time from remission to relapse is also usually recorded in the number of days. In all cases, a discrete random variable is the most appropriate model to fit the data.

* Tel.: +39 02 503 21524; fax: +39 02 503 21505.
E-mail address: alessandro.barbiero@unimi.it.

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1.1. Discrete analogues of continuous random variables

Discrete distributions defined on (positive) integers can be derived from continuous distributions defined on $\mathbb{R}$ ($\mathbb{R}^+$) according to several methods.

In [32,30], the following technique was formalized. If the underlying continuous phenomenon X has the survival function (sf) $S(x) = P(X \geq x)$, the probability mass function (pmf) of its discrete analogue X_d can be derived as

$$\phi(x) = P(X_d = x) = P(x \leq X < x+1) = S(x) - S(x+1); \quad x \in \mathbb{Z}. \tag{1}$$

The first and easiest example in this approach is the geometric distribution with pmf

$$\phi(x) = \theta^x - \theta^{x+1}; \quad x = 0, 1, 2, \ldots$$

which is derived by discretizing the exponential distribution with sf

$$S(x) = \exp(-\lambda x); \quad \lambda, x > 0$$

with $\theta = \exp(-\lambda)$ and $0 < \theta < 1$. Similarly, the negative binomial can be shown to be the discrete analogue of the Gamma distribution. Following this straightforward approach, Nakagawa and Osaki [29] had already obtained the first type discrete Weibull distribution; Roy [31] derived the discrete Rayleigh, Krishna and Singh [25] the discrete Burr, Chakraborty and Chakravarty [10] the discrete Gamma, and so on.

Kemp [19] proposed the following discretization method. A discrete counterpart X_d of a continuous r.v. X with probability density function (pdf) $f(x)$ is derived by computing its pmf as:

$$\phi(x) = P(X_d = x) = \frac{f(x)}{\sum_{x \in \mathbb{Z}} f(x)}, \quad x \in \mathbb{Z}. \tag{2}$$

Kemp discussed the discrete normal r.v. derived by applying this procedure, which is obviously different from the homonym proposed by Roy [30]. Similarly, a discrete half-normal is derived in [20], and a discrete skew Laplace in [23].

Recently, a quite different discretization method was proposed in [2], where a formula for the pmf $\phi(x)$ of a T–X family of discrete distributions is derived:

$$\phi(x) = G(x) - G(x-1) = R\{-\log(1 - F(x))\} - R\{-\log(1 - F(x-1))\}$$

where $G(x) = R\{-\log(1 - F(x))\}$, with $R(t)$ the cumulative distribution function (cdf) of a r.v. T defined on $(0, +\infty)$, and $F(x)$ the cdf of X. In [2], other existing and more specific discretization methods are recalled.

Thus far, although much attention has been paid to deriving discrete models from positive continuous distributions, little interest has been shown in discretizing continuous distributions defined on the whole set $\mathbb{R}$; perhaps, the only exceptions are represented by the discrete normal distribution introduced by Roy [30] and the discrete Laplace distribution by Kozubowski and Inusah [23]. Discrete r.v., defined on the whole set $\mathbb{Z}$, can be usefully used, for example, for modeling differences in count variables [8]. Actually, the Skellam distribution [14,34,35] is the best-known discrete model available for this type of data, and is itself defined as the difference between two independent Poisson.

In this work, the focus is on the skew Laplace distribution and the aim is to define an alternative discrete counterpart.

1.2. Continuous skew Laplace distribution

A continuous r.v. is commonly said to follow the skew Laplace distribution with parameters κ and σ if its pdf is given by:

$$f(x; \kappa, \sigma) = \frac{1}{\sigma} \frac{\kappa}{1+\kappa^2} \begin{cases} \exp\left(-\frac{\kappa}{\sigma} x\right) & \text{if } x \geq 0 \\ \exp\left(\frac{1}{\kappa\sigma} x\right) & \text{if } x < 0 \end{cases} \tag{3}$$