Statistics and Probability Letters 143 (2018) 17–27

Contents lists available at ScienceDirect

Statistics and Probability Letters

journal homepage: www.elsevier.com/locate/stapro

A population evolution model and its applications to random networks

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ARTICLE INFO

Article history:
Received 11 December 2017
Received in revised form 9 May 2018
Accepted 16 July 2018
Available online 26 July 2018

MSC:
05C80
60G42
60J10

Keywords:
Population evolution
Score
Asymptotic distribution
Random graph
Preferential attachment
Scale free

ABSTRACT

A general population evolution model is considered. Any individual of the population is characterized by its score. Certain general conditions are assumed concerning the number of the individuals and their scores. Asymptotic theorems are obtained for the number of individuals having a fixed score. Then it is proved that the score distribution is scale free. The result is applied to obtain the weight distribution of cliques in a random graph evolution model.

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1. Introduction

One of the most famous models in network theory is the preferential attachment model. It was introduced by Barabási and Albert (1999) to describe real-life networks such as the WWW or social and biological networks. In Barabási and Albert (1999) it was proved that the preferential attachment model leads to a scale-free random graph (for a rigorous mathematical proof see Bollobás B. Riordan et al. (2001)). A random graph is called scale-free if it has a power law (asymptotic) degree distribution. Following the paper of Barabási and Albert (1999) several versions of the preferential attachment model were proposed. For those models and for the general theory of random graphs one can consult monographs (Durrett, 2007; Janson et al., 2000; van der Hofstad, 2017).

In Ostroumova et al. (2013) a general graph evolution scheme was presented which covers several preferential attachment type models. They defined a PA-class which covers the original preferential attachment model, the Holme–Kim model, the random Apollonian network, and the Buckley–Osthus model. Then they proved that the PA-class model leads to a scale-free graph.

In this paper we present a further generalization of the model introduced by Ostroumova et al. (2013). It is well-known that population evolution models and random graph evolution models are closely related. Therefore we introduce a general population evolution model which can be applied to describe the evolution of certain random graphs. We consider the evolution of a population where any individual is characterized by its score. We call it Model S. During the evolution both

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https://doi.org/10.1016/j.spl.2018.07.012
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the size of the population and the scores of the individuals can be increased. Let $X_n(s)$ denote the number of individuals having score s at time n. First we describe the behaviour of the expectation $\mathbb{E}[X_n(s)]$, see Theorem 1. Then in Theorem 2 and Corollary 1 we obtain that

$$\lim_{n\to\infty} \frac{X_n(s)}{n} = c(s)$$

almost surely and

$$c(s) \sim C_0 s^{-1-\frac{1}{a}}$$

as $s \to \infty$, where constant a describes the growth rate of the score and C_0 is an appropriate constant depending on the parameters of the model. So the score distribution is scale-free. Our results generalize those of Ostroumova et al. (2013). To obtain Theorems 1 and 2 we apply the methods presented in Ostroumova et al. (2013). We give detailed mathematical proofs in Appendices A and B.

Then we apply our results to a random graph which is based on N-interactions. The 3-interactions model was introduced in Backhausz and Móri (2012) (see also Backhausz and Móri, 2014). The general N-interactions model was introduced and studied in Fazekas and Porvázsnyik (2013, 2016b, a). That model incorporates the preferential attachment rule and the uniform choice of vertices. In that model the vertices and the cliques possess certain weights. In this paper we obtain that in the N-interactions model the weight distribution of the cliques is a power law, see Theorem 3. This theorem generalizes the results of Fazekas I. Noszály Cs and Perecsényi (2015) where the case $N = 3$ was covered.

2. Model S and its asymptotic behaviour

Our aim is to describe the evolution of a population. For the sake of brevity, the evolution procedure defined below till Eq. (2.3) will be called Model S. At time $n = 0$ there exist maximum t members of the population. At each time $n = 1, 2, \ldots$ maximum t individuals are born. Any member of the population is characterized by its score. The value of the score is a positive integer. At birth the score of any individual is u with high probability. Here $u \geq 1$ is a fixed integer. More precisely the score of a new individual is at least u and at time n

$$\mathbb{P}\{\text{the score of a new individual} > u\} = \mathrm{O}\left(\frac{1}{n}\right). \tag{2.1}$$

The score of the individual i at time n is denoted by $S_n(i)$. Let $\mathcal{F}_n$ denote the past of the population up to time n. The evolution of the score of any given individual i is described by the following rules

$$\begin{aligned}
\mathbb{P}\{S_{n+1}(i) = S_n(i) + 1 | \mathcal{F}_n\} &= a\frac{S_n(i)}{n} + b\frac{1}{n} + \mathrm{O}\left(\left(\frac{S_n(i)}{n}\right)^2\right), \\
\mathbb{P}\{S_{n+1}(i) = S_n(i) | \mathcal{F}_n\} &= 1 - a\frac{S_n(i)}{n} - b\frac{1}{n} + \mathrm{O}\left(\left(\frac{S_n(i)}{n}\right)^2\right), \\
\mathbb{P}\{S_{n+1}(i) > S_n(i) + 1 | \mathcal{F}_n\} &= \mathrm{O}\left(\left(\frac{S_n(i)}{n}\right)^2\right),
\end{aligned} \tag{2.2}$$

where a and b are fixed non-negative numbers. So at each step the score of any member of the population is increased by 1 or 0, higher increase is of low probability. Assume also that the cumulated increase of the scores is at most t at each time step. Denote by ξ_n the number of new individuals at time n. Assume that

$$\mathbb{E}[\xi_n] = m + \mathrm{O}\left(\frac{1}{n}\right) \tag{2.3}$$

where $m > 0$.

Let $X_n(s)$ denote the number of individuals having score s at time n. Let $\Theta(s)$ denote a quantity with $|\Theta(s)| < s$. (e.g. the functions $s/2$ and $\ln(1+s)$ are $\Theta(s)$ for $s > 0$; moreover, for fixed $0 < p < q$ we have $s^p = \Theta(s^q)$ for $s > 1$.)

The first theorem shows that the expectation of the score distribution is scale-free.

Theorem 1. *Suppose that the conditions of Model S are satisfied and $a > 0$. Then for any fixed $s = u, u+1, u+2, \ldots$ we have*

$$\mathbb{E}[X_n(s)] = c(s)\left(n + \Theta\left(K s^{2+\frac{1}{a}}\right)\right) \tag{2.4}$$

for all n, where K is a fixed finite constant,

$$c(s) = \frac{\Gamma\left(s + \frac{b}{a}\right)\Gamma\left(u + \frac{b+1}{a}\right)}{a\Gamma\left(s + \frac{b+a+1}{a}\right)\Gamma\left(u + \frac{b}{a}\right)} m \tag{2.5}$$

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