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Fractional Lévy Cox–Ingersoll–Ross and Jacobi processes

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ABSTRACT

We prove a general Picard–Lindelöf-type framework for stochastic differential equations driven by Mandelbrot–Van Ness fractional Lévy processes. This allows us to derive the existence of a fractional Lévy Cox–Ingersoll–Ross and Jacobi model with almost surely positive, respectively bounded, sample paths.

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1. Introduction

Mandelbrot–Van Ness fractional Lévy processes (MvN-flps) have initially been introduced by Marquardt (2006) while their conditional distributions have been analyzed in Fink (2016). Additionally, Fink and Klüppelberg (2011) considered MvN-flp-driven stochastic differential equations (sdes) and constructed explicit solutions based on the general idea of Doss (1977), Lyons (1994), Zähle (1998) and Buchmann and Klüppelberg (2006). However, as has been discussed in Section 5.2 of Fink and Klüppelberg (2011), the theory therein only covers Cox–Ingersoll–Ross (CIR) sdes like

$$dX_t = -\kappa X_t dt + \sigma \sqrt{X_t} dL_t^d \quad \text{or} \quad dX_t = -\kappa X_t dt + \sigma \sqrt{|X_t|} dL_t^d$$

which is not suitable for, e.g., volatility modeling when aiming for a fractional version of the classical Heston setup (cf. Heston (1993)). Therefore, we would like to obtain an existence and uniqueness result regarding a strictly positive solution of the general MvN-flp CIR sde given by

$$dX_t = \kappa(t)(\theta(t) - X_t)dt + \sigma(t)\sqrt{X_t}dL_t^d, \quad t \in [0, T], \quad X_0 = x > 0. \quad (1)$$

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For a fractional Brownian motion (fBm) as a driving process, this has recently been solved by [Schlüchtermann and Yang \(2016\)](#). In particular, due to the fact that fBm has zero quadratic variation, only $\kappa(\cdot)\theta(\cdot) > 0$ is needed to ensure positivity of a solution – similar to the result which we will obtain below. Additionally and of interest for stochastic correlation models we shall consider a fractional version of the (quite similar) Jacobi sde as well, i.e.

$$dX_t = \kappa(t)(\theta(t) - X_t)dt + \sigma(t)\sqrt{X_t}\sqrt{1 - X_t}dL_t^d, \quad t \in [0, T], \quad X_0 = x \in (0, 1). \quad (2)$$

Throughout the paper, we will work on a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with a given square-integrable MvN-flp $L^d = (L_t^d)_{t \in \mathbb{R}}$, $d \in (0, 1/2)$, without Gaussian part in the sense of [Marquardt \(2006\)](#). In particular, only the long memory case is included as paths of L^d are a.s. Hölder continuous up until d (cf. Theorem 4.3 (i) of [Marquardt \(2006\)](#)). Additionally, to ensure the existence of integrals, only MvN-flps with bounded p -variation for $1 \leq p < 2$, similar to [Fink and Klüppelberg \(2011\)](#), are considered. Therefore, from now on, integration shall be understood in the pathwise Riemann–Stieltjes sense (cf. [Young \(1936\)](#)).

Section 2 shall provide a general framework for MvN-flp driven sdes possibly allowing for time-dependent coefficient functions and therefore extending the setup of [Lyons \(1994\)](#). In Section 3 we shall use these results to prove that suitable solutions to (1) and (2) exist a.s. in the pathwise sense. A brief simulation study closes the paper.

2. A general Picard–Lindelöf-type framework

In this section, we want to prove a general Picard–Lindelöf-type existence and uniqueness result for MvN-flp driven sdes on compact time sets. In order to do that, we need a Banach space for potential solutions to live in: For $1 \leq p < 2$ and $a < b$, let $\mathfrak{W}_p^{\text{con}}([a, b])$ be the set of all continuous functions f on $[a, b]$ with bounded p -variation $v_p(f, [a, b])$, where

$$v_p(f, [a, b]) := \sup_{\kappa} \sum_{i=1}^n |f(x_i) - f(x_{i-1})|^p$$

with the sup taken over all grids κ of $[a, b]$. Applying Minkowski's inequality shows that by pointwise addition and scalar multiplication $\mathfrak{W}_p^{\text{con}}([a, b])$ becomes a $\mathbb{R}$ -vector space and $(v_p(\cdot, [a, b]))^{\frac{1}{p}}$ is a seminorm on $\mathfrak{W}_p^{\text{con}}([a, b])$ since, obviously, we have $v_p(f, [a, b]) = 0$ for every constant function f . To overcome this problem and to obtain a normed vector space we could consider the quotient space

$$\mathfrak{W}_p^{\text{con}}([a, b]) / \text{const}([a, b])$$

where $\text{const}([a, b])$ is the vector space of all constant functions on $[a, b]$. This approach however causes problems when considering integral equations – solutions would only be unique up to a.s. constant shifts. Instead we define similar to [Chistyakov and Galkin \(1998\)](#) an actual norm on $\mathfrak{W}_p^{\text{con}}([a, b])$ by

$$\|\cdot\|_p := \|\cdot\|_p^{[a, b]} := \|\cdot\|_{\text{sup}}^{[a, b]} + (v_p(\cdot, [a, b]))^{\frac{1}{p}}$$

where we will suppress the $[a, b]$ in the notation when the interval is clear. Proposition 7.2 of [Chistyakov and Galkin \(1998\)](#) now implies that $(\mathfrak{W}_p^{\text{con}}([a, b]), \|\cdot\|_p)$ is a Banach space. For the considerations to come, we shall need the following technical lemma.

Lemma 2.1. Let $[a, b]$ be a compact interval, $g \in \mathfrak{W}_p^{\text{con}}([a, b])$ and $f \in \mathfrak{W}_q^{\text{con}}([a, b])$ where $q > 0$ and $p \geq 1$ with $p^{-1} + q^{-1} > 1$. For all $x \in [a, b]$ we define $\phi(x) := \int_a^x f dg$. Then $\phi \in \mathfrak{W}_p^{\text{con}}([a, b])$. Moreover we have with ζ denoting the Riemann zeta function

$$v_p(\phi, [a, b]) \leq 2^p \left(\left(1 + \zeta \left(\frac{1}{p} + \frac{1}{q} \right) \right)^p v_q(f, [a, b])^{\frac{p}{q}} + \|f\|_{\text{sup}}^p v_p(g, [a, b]) \right).$$

Proof. The first part follows from Theorem A.3 of [Fink and Klüppelberg \(2011\)](#). For the inequality, we recall that for $p \geq 1$ and $x, y \in \mathbb{R}$ we have $|x + y|^p \leq 2^p(|x|^p + |y|^p)$ and therefore calculate for $z_i \in [x_{i-1}, x_i] \subseteq [a, b]$

$$\begin{aligned} v_p(\phi, [a, b]) &= \sup_{\kappa} \sum_{i=1}^n \left| \int_{x_{i-1}}^{x_i} f dg \right|^p = \sup_{\kappa} \sum_{i=1}^n \left| \int_{x_{i-1}}^{x_i} f dg - f(z_i)[g(x_i) - g(x_{i-1})] + f(z_i)[g(x_i) - g(x_{i-1})] \right|^p \\ &\leq \sup_{\kappa} \sum_{i=1}^n \left(\left| \int_{x_{i-1}}^{x_i} (f - f(z_i)) dg \right| + |f(z_i)| |g(x_i) - g(x_{i-1})| \right)^p \\ &\leq 2^p \sup_{\kappa} \sum_{i=1}^n \left(\left| \int_{x_{i-1}}^{x_i} (f - f(z_i)) dg \right|^p + |f(z_i)|^p |g(x_i) - g(x_{i-1})|^p \right). \end{aligned}$$

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