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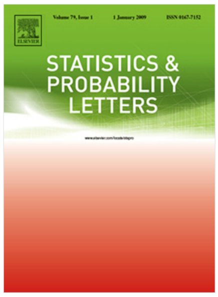

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A uniform L^1 law of large numbers for functions of i.i.d. random variables that are translated by a consistent estimator

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Abstract

We develop a new L^1 law of large numbers where the i-th summand is given by a function $h(\cdot)$ evaluated at $X_i - \theta_n$, and where $\theta_n \doteq \theta_n(X_1, X_2, \dots, X_n)$ is an estimator converging in probability to some parameter $\theta \in \mathbb{R}$. Under broad technical conditions, the convergence is shown to hold uniformly in the set of estimators interpolating between θ and another consistent estimator $\theta_n^\star$. Our main contribution is the treatment of the case where $|h|$ blows up at 0, which is not covered by standard uniform laws of large numbers.

Keywords: uniform law of large numbers, Taylor expansion, M-estimators, score function
2010 MSC: 60F25

1. Introduction

Let $X_1, X_2, X_3, \dots$ be a sequence of i.i.d. random variables and consider the statistic $T_n(\theta_n^\star)$ where the random variable

$$T_n(\theta) \doteq T_n(X_1, X_2, \dots, X_n; \theta) : \Omega \to \mathbb{R}$$

depends on an unknown parameter $\theta \in \mathbb{R}$ for which we have a consistent sequence of estimators $\theta_n^\star \doteq \theta_n^\star(X_1, X_2, \dots, X_n)$. Assume further that the following first-order Taylor expansion is valid :

$$T_n(\theta_n^\star) = T_n(\theta) + (\theta_n^\star - \theta) \int_0^1 T_n'(\theta + v(\theta_n^\star - \theta)) dv, \tag{1.1}$$

where

$$T_n'(t) = \frac{1}{n} \sum_{i=1}^n \mathbf{1}_{\{X_i \neq t\}} h(X_i - t), \tag{1.2}$$

and where $h : \mathbb{R}\backslash\{0\} \to \mathbb{R}$ is a measurable function (possibly nonlinear). In statistics, one is often interested in knowing if estimating a parameter (θ here) has an impact on the asymptotic law of a given statistic. See for example the interesting results of de Wet and Randles (1987) in the context of limiting χ^2 U and V statistics. Equations (1.1) and (1.2) provide a natural setting for studying the question of whether or not $T_n(\theta_n^\star) - T_n(\theta) \to 0$ whenever $\theta_n^\star \to \theta$, as $n \to \infty$.

Given some regularity conditions on the behavior of $h(\cdot)$ around the origin and in its tails, proving the convergence to $\mathbb{E}[h(X_1 - \theta)]$, in probability say, of the integral on the right-hand side of (1.1) is often possible under weak assumptions by adapting standard uniform laws of large numbers. For instance, one can use (Ferguson, 1996, Theorem 16 (a)), which was introduced by LeCam (1953) and Rubin (1956). One can also use entropy conditions: see, e.g., (van de Geer, 2000, Chapter 3) and (van der Vaart and Wellner, 1996, Section 2.4). Some of these theorems go back to or evolved from the works of Blum (1955), Dehardt (1971), Vapnik and Červonenkis (1971, 1981), Giné and Zinn (1984), Pollard (1984) and Talagrand (1987). For extensive notes on the origins of the entropy conditions, we refer the interested reader to (van de Geer, 2000, Section 3.8) and (Pollard, 1984, pp. 36–38).

However, when $|h|$ blows up at 0, namely when $\limsup_{x\to 0} |h(x)| = \infty$, these results are not applicable because the enveloppe function $h^{\sup}(x) \doteq \sup_{t:|t-\theta|<\delta} \mathbf{1}_{\{x\neq t\}} |h(x - t)|$ is infinite in any small enough neighborhood of θ and, in particular, $h^{\sup}(X_1)$ is not integrable for the outer measure.

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