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Long time behavior for stochastic Burgers equations with jump noises[☆]

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ABSTRACT

In this paper, we consider stochastic Burgers equations driven by compensated Poisson random measures. Under some appropriate conditions, we investigate the exponential stability on the solutions of the equations. Examples are presented to illustrate the applications of the results.

1. Introduction

The Burgers equation proposed in Burgers (1973) has played an important role in fluid dynamics. It has the following form:

$$\frac{\partial X(t,\xi)}{\partial t} = \mu \frac{\partial^2 X(t,\xi)}{\partial \xi^2} + X(t,\xi)\frac{\partial X(t,\xi)}{\partial \xi}, \tag{1.1}$$

where $\frac{\partial X(t,\xi)}{\partial t}$ is the velocity field and μ is the viscosity. The equation represents a simplified model for describing the interaction of dissipative and non-linear inertial terms in the fluid motion (a detailed discussion of the physics aspects of the equation can be found in Lighthill (1956)). Due to the existence of some extra forces such as chaotic phenomena and turbulence, some stochastic versions of the Burgers equation now have been developed by many authors (see, e.g., Bertini et al., 1994, Da Prato et al., 1994, Feng et al., 2017, Khanin et al., 2000, Hosokawa and Yamamoto, 1975, Jeng, 1969, Schulze-Halberg, 2015, Liu, 1997, Truman and Zhao, 1998 and references therein). Liu (1997) studies the long time behavior for stochastic Burgers equations driven by Gaussian white noises. Via imaginary-Brownian-time-Brownian angle process, Allouba (2003) investigates L-KS equations in all spatial dimensions. Allouba (2006, 2015) and Allouba and Xiao

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(2017) formulate and obtain sharp regularity results for many SPDEs of L-KS type in the presence of the rough driving space–time white noise in dimensions $1 \le d \le 3$. Wang et al. (2014) investigate asymptotic stabilities of nonlocal stochastic K-S equation with white noises. When considering the noise suitable for modeling the random perturbation source, we observe that an important attractive feature of the pure jump diffusive component of a Poisson random measure is its ability to capture rare events with low frequency and sudden occurrence (Protter and Talay, 1997). This has led to considerable interest in stochastic partial differential equations (SPDEs) with jump noises (see, e.g., Albeverio et al., 1998, Bo et al., 2007, Dong and Xu, 2007 and Truman and Wu, 2003). Dong and Xu (2007) discuss 1-dimensional stochastic Burgers equations driven by a compensated Poisson random measure. Though several researchers have investigated the stochastic equations driven by jump noises, no one to our knowledge has studied the asymptotic behavior of solutions for stochastic Burgers equations driven by compensated Poisson random measures. In this paper, we consider the following 1-dimensional stochastic Burgers equation with jump noise,

$$\begin{cases} \dfrac{\partial X(t,\xi)}{\partial t} = \mu \dfrac{\partial^2 X(t,\xi)}{\partial \xi^2} + X(t,\xi)\dfrac{\partial X(t,\xi)}{\partial \xi} + \displaystyle\int_Z \sigma\,(t-, X(t-,\xi), z)\,\widetilde{N}(\mathrm{d}z,\mathrm{d}t), \quad t > 0, \\ X(t,0) = X(t,1) = 0, \quad t \ge 0, \\ X(0,\xi) = x(\xi), \quad \xi \in D := [0,1], \end{cases} \tag{1.2}$$

where μ is a viscosity constant, and the jump noise $\widetilde{N}(\mathrm{d}z,\mathrm{d}t)$ is detailed below. The long term behavior of its solution, more specifically, the exponential stability will be studied. Moreover, there have been some increasing interests on the stochastic equations with jump noises. For instance, Bo et al. (2007) investigate the existence and uniqueness of the weak solution for 1-dimensional stochastic Kuramoto–Sivashinsky equations by developing the noise to be a compensated Poisson random measure. They show that an invariant measure of the equation indeed exists under some appropriate assumptions. Here we consider a stochastic version of Burgers equation, in which the noise is formulated by a compensated Poisson random measure with periodic boundary conditions.

The rest of the paper is organized as follows: Section 2 gives some preliminaries and main hypotheses. Section 3 is devoted to the discussion on the long time behavior of the solution of stochastic equations.

2. Preliminaries and hypotheses

We begin with some notations and functional spaces, which will be used frequently in the following sections. Assume that $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\ge 0}, \mathbf{P})$ is a complete filtered probability space satisfying the usual condition. Let $N: \mathcal{B}(Z) \times R_+ \times \Omega \to \mathbf{N} \cup \{0\}$ be a Poisson random measure with the characteristic measure $\Pi(\cdot)$ on some measurable space $(Z, \mathcal{B}(Z))$ satisfying $\Pi(Z) < \infty$. Denote $\widetilde{N}(\mathrm{d}z,\mathrm{d}t) := N(\mathrm{d}z,\mathrm{d}t) - \Pi(\mathrm{d}z)\mathrm{d}t$ be a compensated Poisson random measure. $\{N((0,t] \times S); (t,S) \in R_+ \times \mathcal{B}(Z)\}$ can be represented by a Z-valued point function $\{\mathrm{p}(t); t \ge 0\}$ with the domain D_{p} as a countable subset of R_+ (see, e.g., Ikeda and Watanabe, 1989). That is,

$$N((0,t] \times S) = \sum_{s \in D_{\mathrm{p}}, s \le t} \mathbf{1}_S(\mathrm{p}(s)), \quad \text{for } t > 0 \text{ and } S \in \mathcal{B}(Z).$$

Recall Eq. (1.2) and let u be a solution. If $\sigma \equiv 0$, it is known that u is periodic. Without loss of generality, define

$$\begin{aligned} H_{\mathrm{per}} &:= \left\{ u \in L^2(D) : \ u \text{ is periodic on } D, \int_0^1 u(x)\mathrm{d}x = 0 \right\}, \\ H^p_{\mathrm{per}} &:= \left\{ u \in W^{2,p}(D) : \ u \text{ is periodic on } D \right\}, \quad \text{for each } p \in \mathbf{N}, \\ \dot{H}^p_{\mathrm{per}} &:= H^p_{\mathrm{per}} \cap H_{\mathrm{per}}, \quad \text{for each } p \in \mathbf{N}, \end{aligned}$$

where $W^{2,p}(D)$ is the standard Sobolev space (Adams, 1992). Denote the norm of H_{per} by $\|u\| = (u,u)^{\frac{1}{2}}$, and the inner product in H_{per} by

$$(u,v) = \int_D u(\xi)v(\xi)\mathrm{d}\xi, \quad u, v \in H_{\mathrm{per}}.$$

Let $D_0 = \mathrm{I}$ be the identity operator on H_{per} and for $i \in \mathbf{N} \cup \{0\}$, set $D_i = \frac{\partial^i}{\partial x^i}$. Set $A = -D_2$ and $V = D(A)$, then A is a positive self-adjoint linear operator on V. Thus there exists an orthonormal basis of H_{per}, $(e_k)_{k=1,2,\ldots}$, which consists of eigenvectors of $A : V \to H_{\mathrm{per}}$ such that $Ae_k = \lambda_k e_k$ for $k = 1, 2, \ldots$ and $0 < \lambda_1 \le \lambda_2 \le \cdots$, with $\lim_{k\to\infty}\lambda_k = +\infty$. The spectral theory of operator A allows us to define A^s as the powers of A as follows (Da Prato and Zabczyk, 1996),

$$D(A^s) = \left\{ u \in H_{\mathrm{per}}; \ \sum_{k=1}^{\infty} \lambda_k^{2s}(u,e_k)^2 < \infty \right\},$$

and

$$A^s u = \sum_{k=1}^{\infty} \lambda_k^s\,(u,e_k)\,e_k, \quad \text{for } u \in D(A^s), \ s \in R,$$