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A note on joint occupation times of spectrally negative Lévy risk processes with tax

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ABSTRACT

In this paper we consider the joint Laplace transform of occupation times over disjoint intervals for spectrally negative Lévy processes with a general loss-carry-forward taxation structure. This tax structure was first introduced by Albrecher and Hipp in their paper in 2007. We obtain representations of the joint Laplace transforms in terms of scale functions and the Lévy measure associated with the driven spectrally negative Lévy processes. Two numerical examples, i.e. a Brownian motion with drift and a compound Poisson model, are provided at the end of this paper and explicit results are presented with discussions.

1. Introduction

Lévy processes are stochastic processes with independent and stationary increments. Spectrally negative Lévy processes (SNLPs) are Lévy processes with no upward jumps, which find many applications in risk theory, mathematical finance and branching processes. In the recent literature of risk theory and mathematical finance, there have been increasing interests in studying the Laplace transforms of occupation times for Lévy processes. For general SNLPs, Laplace transforms of occupation times were studied in Landriault et al. (2011) and Loeffen et al. (2014), by adopting different approximation schemes. Quite recently, the joint Laplace transforms of occupation times under Lévy processes have been attracting much research attention. For instance, Li and Zhou (2013) considered joint occupation times under general time-homogeneous diffusion processes. Further more, Li and Zhou (2014) derived the joint Laplace transforms of occupation times over disjoint intervals under SNLPs, by adopting a fairly new approach. Some recent papers considering SNLPs include Yin and Yuen (2014) and Li et al. (2017).

The so-called loss-carry-forward taxation system (in a simplified version) was first introduced into a compound Poisson process with drift by Albrecher and Hipp (2007). Meanwhile, Kyprianou and Zhou (2009) introduced a very general taxation structure into the Lévy framework. Results regarding stochastic processes with loss-carry-forward taxation can be found in Wang and Hu (2012), Wang et al. (2011), Ming et al. (2010), Albrecher et al. (2008) and the references therein.

This paper aims to study the impact of a loss-carry-forward taxation system on the joint Laplace transforms of occupation times in SNLPs. It is motivated by the increasing role of occupation times on managing risks in risk theory. For $0 < a < b$, the occupation times of the surplus process being in intervals $(0, a)$ and (a, b) prior to ruin can be used to evaluate the

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performance of an insurance portfolio as well as monitoring the time an insurer's surplus remaining at critically low levels, which may help to measure the solvency risk. By incorporating taxes into the surplus models will enable us to better examine the above mentioned risks in a more real-life related environment. The obtained new occupation-time functionals are of much interest on both theoretical and practical aspects.

2. Preliminary identities for SNLPs

In this section, we shall provide some preliminary scale function related results for SNLPs. Then, we shall present our model, i.e., an SNLP embedded with a general loss-carry-forward taxation system. Some existing fluctuational and distributional identities on our taxed model will also be given in this section.

2.1. SNLPs without tax

Let process $X = \{X_t; t \geq 0\}$ be an SNLP defined on a filtered probability space $(\Omega, \{\mathcal{F}_t; t \geq 0\}, P)$. We exclude the case of X being the negative of a subordinator. Denote by $\mathbb{P}_x$ the probability law of X given $X_0 = x$, and by $\mathbb{E}_x$ its corresponding expectation operator. The Laplace exponent of X is defined as

$$\psi(\theta) = \log \mathbb{E}_x[e^{\theta(X_1 - x)}],$$

which is finite at least for $\theta \in [0, \infty)$, where it is strictly convex and infinitely differentiable. The scale functions $\{W^{(q)}; q \geq 0\}$ of X are defined such that for each $q \geq 0$, $W^{(q)}: [0, \infty) \to [0, \infty)$ is the unique strictly increasing and continuous function with a Laplace transform satisfying $\int_0^\infty e^{-\lambda x} W^{(q)}(x) \mathrm{d}x = \frac{1}{\psi(\lambda) - q}$, $\lambda > \Phi(q)$, where $\Phi(q)$ is the larger solution of the equation $\psi(\lambda) = q$ (there are at most two). Let $W^{(q)'}(x)$ be its density and we let $W^{(q)}(x) = 0$ for $x < 0$.

Define the first up-crossing and down-crossing times of X as follows,

$$T_b^+ = \inf\{t \geq 0;\ X_t \geq b\}, \qquad T_a^- = \inf\{t \geq 0;\ X_t < a\}$$

with the convention that $\inf \phi = \infty$. In addition, define for $q \geq 0$,

$$Z^{(q)}(x) = \begin{cases} 1 + q \displaystyle\int_0^x W^{(q)}(y) dy, & x \geq 0, \\ 1, & x < 0. \end{cases}$$

According to Kyprianou (2014) we know that, for $x \leq b$,

$$\mathbb{E}_x\left[e^{-qT_b^+}; T_b^+ < T_0^-\right] = \frac{W^{(q)}(x)}{W^{(q)}(b)}, \quad \mathbb{E}_x\left[e^{-qT_0^-}; T_0^- < T_b^+\right] = Z^{(q)}(x) - \frac{W^{(q)}(x)}{W^{(q)}(b)} Z^{(q)}(b).$$

Furthermore, it has been verified by Li and Zhou (2014) that, for $q_1, q_2 \geq 0$, $a, x \in [0, b]$ and $b \in (0, \infty)$, we have,

$$\begin{aligned} g_1(x, b) &:= \mathbb{E}_x\left[\exp\left\{-q_1 \int_0^{T_0^-} \mathbf{1}_{(0,a)}(X_s) ds - q_2 \int_0^{T_0^-} \mathbf{1}_{(a,b)}(X_s) ds\right\}; T_0^- < T_b^+\right] \\ &= Z_a^{(q_1,q_2)}(x) - \frac{W_a^{(q_1,q_2)}(x) Z_a^{(q_1,q_2)}(b)}{W_a^{(q_1,q_2)}(b)}, \end{aligned} \tag{2.1}$$

$$\begin{aligned} g_2(x, b) &:= \mathbb{E}_x\left[\exp\left\{-q_1 \int_0^{T_b^+} \mathbf{1}_{(0,a)}(X_s) ds - q_2 \int_0^{T_b^+} \mathbf{1}_{(a,b)}(X_s) ds\right\}; T_0^- > T_b^+\right] \\ &= \frac{W_a^{(q_1,q_2)}(x)}{W_a^{(q_1,q_2)}(b)}. \end{aligned} \tag{2.2}$$

Here, for $q_1, q_2 \geq 0$ and $0 \leq a \leq x$,

$$W_a^{(q_1,q_2)}(x) := W^{(q_1)}(x) - (q_1 - q_2) \int_a^x W^{(q_1)}(y) W^{(q_2)}(x - y) dy,$$

$$Z_a^{(q_1,q_2)}(x) := Z^{(q_1)}(x) - (q_1 - q_2) \int_a^x Z^{(q_1)}(y) W^{(q_2)}(x - y) dy.$$

2.2. SNLPs with tax

Following the ideas in Albrecher and Hipp (2007) and Kyprianou and Zhou (2009), we are interested in how the loss-carry-forward tax payments will affect the quantitative behavior of the driven SNLP. Assume that the cumulative tax payments by time t are given by

$$\int_0^t \gamma(S_w^X) \mathrm{d}S_w^X,$$