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A revisit to asymptotic ruin probabilities for a bidimensional renewal risk model

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ABSTRACT

Recently, Yang and Li (2014) studied a bidimensional renewal risk model with constant force of interest and dependent subexponential claims. Under the special Farlie–Gumbel–Morgenstern dependence structure and a technical moment condition on the claim-number process, they derived an asymptotic expansion for the finite-time ruin probability. In this paper, we show that their result can be extended to a much more general dependence structure without any extra condition on the renewal claim-number process. We also give some asymptotic expansions for the corresponding infinite-time ruin probability within the scope of extended regular variation.

1. Introduction and main results

In the past decade, as natural extensions of the classical renewal risk model, more and more bidimensional risk models have been proposed and deeply investigated. See Li et al. (2007), Chen et al. (2011), Zhang and Wang (2012), Chen et al. (2013a, b), Hu and Jiang (2013), Yang and Li (2014), Li and Yang (2015), and Li (2017), among others.

Most recently, Yang and Li (2014) considered a bidimensional renewal risk model with constant force of interest and dependent subexponential claims. Their model can be quantitatively expressed as follows:

$$\begin{pmatrix} U_1(t) \\ U_2(t) \end{pmatrix} = \begin{pmatrix} xe^{rt} \\ ye^{rt} \end{pmatrix} + \begin{pmatrix} \int_{0-}^{t} e^{r(t-s)} C_1(ds) \\ \int_{0-}^{t} e^{r(t-s)} C_2(ds) \end{pmatrix} - \begin{pmatrix} \sum_{i=1}^{N(t)} X_i e^{r(t-\tau_i)} \\ \sum_{i=1}^{N(t)} Y_i e^{r(t-\tau_i)} \end{pmatrix}, \qquad t \geq 0, \tag{1.1}$$

where $\{(U_1(t), U_2(t)); t \geq 0\}$ denotes the bidimensional surplus process of an insurer with two lines of businesses, (x, y) the vector of the initial surpluses, $r \geq 0$ the constant force of interest, $\{(C_1(t), C_2(t)); t \geq 0\}$ the premium accumulation process with the nondecreasing and right continuous paths satisfying $(C_1(0), C_2(0)) = (0, 0)$, and $\{(X_i, Y_i); i \geq 1\}$ the sequence of claim size vectors whose common arrival times $\tau_1, \tau_2, \ldots$ constitute a renewal claim-number process $\{N(t); t \geq 0\}$ with

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finite renewal function $\lambda(t) = \mathbb{E}N(t) = \sum_{i=1}^{\infty}\mathbb{P}(\tau_i \le t)$. It is assumed that $\{(X_i, Y_i); i \ge 1\}$ is a sequence of independent and identically distributed (i.i.d.) random vectors with generic vector (X, Y) whose marginal distribution functions are $F = 1 - \overline{F}$ on $[0, \infty)$ and G on $[0, \infty)$, respectively, and that $\{(X_i, Y_i); i \ge 1\}$, $\{(C_1(t), C_2(t)); t \ge 0\}$ and $\{N(t); t \ge 0\}$ are mutually independent.

Define the finite-time and infinite-time ruin probabilities of risk model (1.1), respectively, as

$$\psi(x, y; T) = \mathbb{P}(T_{\max} \le T | (U_1(0), U_2(0)) = (x, y)), \qquad T > 0,$$

and

$$\psi(x, y) = \lim_{T\to\infty} \psi(x, y; T) = \mathbb{P}(T_{\max} < \infty | (U_1(0), U_2(0)) = (x, y)),$$

where

$$T_{\max} = \inf\{t > 0 : \max\{U_1(t), U_2(t)\} < 0\}$$

denotes the ruin time with $\inf\emptyset = \infty$ by convention.

By definition, a distribution function V on $[0, \infty)$ is said to belong to the subexponential class, written as $V \in \mathcal{S}$, if $\overline{V}(x) > 0$ for all $x \ge 0$ and the relation

$$\lim_{x\to\infty} \frac{\overline{V^{n*}}(x)}{\overline{V}(x)} = n$$

holds for all (or, equivalently, for some) $n \ge 2$, where V^{n*} is the n-fold convolution of V with itself. Additionally, in what follows all limit relations hold as $(x, y) \to (\infty, \infty)$ unless otherwise stated. For two positive bivariate functions $f(\cdot, \cdot)$ and $g(\cdot, \cdot)$, we write $f \lesssim g$ or $g \gtrsim f$ if $\limsup f/g \le 1$, write $f \sim g$ if both $f \lesssim g$ and $f \gtrsim g$, and write $f \asymp g$ if $0 < \liminf f/g \le \limsup f/g < \infty$.

Under a moment condition on $\{N(t); t \ge 0\}$ and a dependence assumption that (X, Y) follows the Farlie–Gumbel–Morgenstern (FGM) distribution, Yang and Li (2014) established the following proposition, which gives a precise asymptotic expansion for $\psi(x, y; T)$ as $(x, y) \to (\infty, \infty)$ with fixed T for subexponential claims.

Proposition 1.1 (*Theorem 2.1 of Yang and Li (2014)*). *Consider the bidimensional risk model (1.1). Let $F \in \mathcal{S}$, $G \in \mathcal{S}$, and (X, Y) follow the FGM distribution given as*

$$\mathbb{P}(X \le x, Y \le y) = F(x)G(y)\left(1 + \theta\overline{F}(x)\overline{G}(y)\right)$$

with $\theta \in (-1, 1]$. Let $T > 0$ such that $\lambda(T) > 0$.

(i) If $\theta \in (-1, 0]$ then

$$\psi(x, y; T) \sim \iint_{\substack{s,t\ge 0\\ s+t\le T}} \left[\overline{F}\left(xe^{r(t+s)}\right)\overline{G}\left(ye^{rt}\right) + \overline{F}\left(xe^{rt}\right)\overline{G}\left(ye^{r(t+s)}\right)\right]\lambda(ds)\lambda(dt)$$
$$+ (1+\theta)\int_{0-}^{T} \overline{F}\left(xe^{rt}\right)\overline{G}\left(ye^{rt}\right)\lambda(dt). \tag{1.2}$$

(ii) If $\theta \in (0, 1]$ and $\mathbb{E}\kappa^{N(T)} < \infty$ for some $\kappa > 1 + \theta$, then relation (1.2) holds.

In this paper, we extend Proposition 1.1 from the special FGM case to a much more general dependence structure between X and Y. Concretely speaking, we assume the following:

Assumption A. There is some $\rho > 0$ such that

$$\mathbb{P}(X > x, Y > y) \sim \rho\overline{F}(x)\overline{G}(y).$$

It is easy to check that many commonly used copulas, including the FGM one, satisfy Assumption A; see Section 2 for details. Moreover, being superior to Proposition 1.1(ii), we establish the asymptotic expansion for $\psi(x, y; T)$ without any extra condition on the claim-number process $\{N(t); t \ge 0\}$.

Our first main result is given in the following theorem, whose proof will be shown in Section 3.

Theorem 1.1. *Consider the bidimensional risk model (1.1). Let $F \in \mathcal{S}$, $G \in \mathcal{S}$, and Assumption A hold. For each $T > 0$ such that $\lambda(T) > 0$, we have*

$$\psi(x, y; T) \sim \iint_{\substack{s,t\ge 0\\ s+t\le T}} \left[\overline{F}\left(xe^{r(t+s)}\right)\overline{G}\left(ye^{rt}\right) + \overline{F}\left(xe^{rt}\right)\overline{G}\left(ye^{r(t+s)}\right)\right]\lambda(ds)\lambda(dt)$$
$$+\rho\int_{0-}^{T} \overline{F}\left(xe^{rt}\right)\overline{G}\left(ye^{rt}\right)\lambda(dt). \tag{1.3}$$