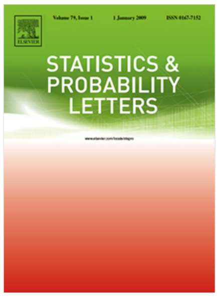
Accepted Manuscript



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PII: S0167-7152(18)30216-5
DOI: https://doi.org/10.1016/j.spl.2018.05.029
Reference: STAPRO 8264

To appear in: *Statistics and Probability Letters*

Received date : 30 December 2017
Revised date : 16 March 2018
Accepted date : 29 May 2018

ON SOME APPLICATIONS OF SOBOLEV FLOWS OF SDES WITH UNBOUNDED DRIFT COEFFICIENTS

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Abstract. We study two applications of spatial Sobolev smoothness of stochastic flows of unique strong solution to stochastic differential equations (SDEs) with irregular drift coefficients. First, we analyse the stochastic transport equation assuming that the drift coefficient is Borel measurable, with spatial linear growth and show that the above equation has a unique Sobolev differentiable weak coefficient for all $t \in [0, T]$ for T small enough. Second, we consider the Kolmogorov equation and obtain a representation of the spatial derivative of its solution v. The latter result is obtained via the martingale representation theorem given in (Elliott and Kohlmann, 1988) and generalises the results in (Elworthy and Li, 1994; Menoukeu-Pamen et al., 2013).

1. Introduction

This paper aims at studying two applications of Sobolev regularity of flows of strong solution to the following SDE

$$\mathrm{d}X_t = b(t, X_t)\mathrm{d}t + \mathrm{d}B_t, \quad 0 \le t \le T, \quad X_0 = x \in \mathbb{R}^d, \tag{1.1}$$

where the drift coefficient $b : [0, T] \times \mathbb{R}^d \longrightarrow \mathbb{R}^d$ is a Borel measurable function satisfying spatial linear growth condition and B_t is a $d-$dimensional Brownian motion on a probability space $(\Omega, \mathcal{F}, P)$. It is known that when b is Lipschitz and satisfies linear growth, then there exists a unique strong solution to the SDE (1.1). Zvonkin (Zvonkin, 1974) shows that if $d = 1$ and b is bounded and measurable, the SDE (1.1) has a unique strong solution. This result was generalised in (Veretennikov, 1979) to the multidimensional case. In the one dimensional case, the authors in (Engelbert and Schmidt, 1989, 1991) show that there exists a unique strong solution to the SDE (1.1) when b is time homogeneous and of spatial linear growth. The time dependent drift coefficient was considered in (Nilssen, 2012), where Malliavin smoothness and Sobolev differentiability of the flows of the solution were also obtained for small time interval. The work (Menoukeu-Pamen et al., 2013) uses Malliavin calculus and white noise analysis to construct a unique strong Malliavin differentiable solution to the SDE (1.1) in multi-dimension. In the recent work (Menoukeu-Pamen and Mohammed, 2016), the authors extended the previous results to the case of irregular and possibly unbounded drift coefficients. In fact, they show that when the drift coefficient b has linear growth with respect to the space variable, there exists a unique strong Malliavin smooth solution to the SDE (1.1). In addition, the solution has a Sobolev differentiable flows. The above results significantly extend the existing ones. Note that when b is only bounded and measurable, the Sobolev smoothness of the unique stochastic flows of (1.1) was obtained in (Mohammed et al., 2015).

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2010 *Mathematics Subject Classification.* 60H10, 60H15, 60H40.

Key words and phrases. Strong solutions of SDE's; irregular drift coefficient; Malliavin calculus; Transport equation; Bismut-Elworthy-Li formula.