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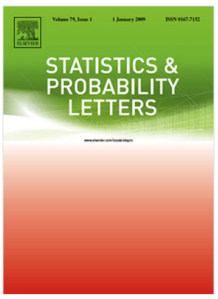

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A Probabilistic Proof for Fourier Inversion Formula

Tak Kwong Wong[a], Sheung Chi Phillip Yam[b]

[a] *Department of Mathematics, The University of Hong Kong, Pokfulam, Hong Kong*
[b] *Department of Statistics, Chinese University of Hong Kong, Shatin, N.T., Hong Kong*

Abstract

The celebrated Fourier inversion formula provides a useful way to re-construct a regular enough, e.g. square-integrable, function via its own Fourier transform. In this article, we give the first probabilistic proof of this classical theorem, even for Euclidean spaces of arbitrary dimension. Particularly, our proof motivates why the one-half weight, for the one-dimensional case in Lemma 1, comes naturally to play due to the inherent spatial symmetry; another similar interpretation can be found in the higher dimensional analogue.

Keywords: Fourier transform, gamma distribution, harmonic analysis, law of large numbers, saddle-point approximation, solid angles.
2010 MSC: 42A38

1. Introduction

Fourier transforms, regarded as a common method to convert problems arisen in spacial domain to that in the frequency domain, are an indispensable tool in mathematical physics and engineering contexts for centuries. A cornerstone result in this theory is the well-known theorem called Fourier inversion theorem/formula (a.k.a. Fourier integral theorem), which leads to an explicit representation of the inverse operator of the Fourier transform. To the best of the authors' knowledge, classical proofs of this important result are all non-probabilistic.

Email addresses: takkwong@maths.hku.hk (Tak Kwong Wong), scpyam@sta.cuhk.edu.hk (Sheung Chi Phillip Yam)