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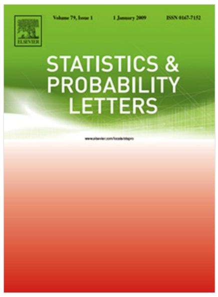

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Closure properties of O-exponential distributions*

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Abstract

Sufficient conditions are found under which randomly stopped sums, randomly stopped maximums, and randomly stopped maximums of sums are distributed according to $\mathcal{O}$-exponential laws. It is supposed that the basic random variables $\{\xi_1, \xi_2, \dots\}$ are real valued and not necessarily identically distributed, whereas the counting random variable η is integer valued, nonnegative, and not degenerate at zero.

Keywords: exponential distribution; O-exponential distribution; randomly stopped sum; randomly stopped maximum; closure property; real-valued random variable.

2010 Mathematics Subject Classification: 60E05; 60G50; 60F10.

1 Introduction

Let $\{\xi_1, \xi_2, \dots\}$ be a sequence of real-valued random variables (r.v.s), identically or nonidentically distributed, and let η be a counting r.v. independent of sequence $\{\xi_1, \xi_2, \dots\}$. As usual, an *r.v. η is called a counting r.v. if it is nonnegative, integer valued, and not degenerate at zero.*

We denote by $S_0 = 0$, $S_n = \xi_1 + \dots + \xi_n$, $n \geqslant 1$, the partial sums, and by $S_\eta = \xi_1 + \dots + \xi_\eta$ the randomly stopped sum of r.v.s $\{\xi_1, \xi_2, \dots\}$. Similarly, let $\xi_{(n)} = \max\{0, \xi_1, \dots, \xi_n\}$, $n \geqslant 1$, $\xi_{(0)} = 0$, and let $\xi_{(\eta)} = \max\{0, \xi_1, \dots, \xi_\eta\}$ be the randomly stopped maximum of r.v.s $\{\xi_1, \xi_2, \dots\}$. Finally, let $S_{(n)} = \max\{S_0, S_1, \dots, S_n\}$, $n \geqslant 0$, and let $S_{(\eta)} = \max\{S_0, S_1, \dots, S_\eta\}$ be the randomly stopped maximum of sums $\{S_0, S_1, S_2, \dots\}$.

The distribution functions (d.f.s) of r.v.s S_η, $\xi_{(\eta)}$, and $S_{(\eta)}$ can be expressed as follows:

$$F_{S_\eta}(x) := \mathbb{P}(S_\eta \leqslant x) = \sum_{n=0}^{\infty} \mathbb{P}(S_n \leqslant x)\mathbb{P}(\eta = n),$$

$$F_{\xi_{(\eta)}}(x) := \mathbb{P}(\xi_{(\eta)} \leqslant x) = \sum_{n=0}^{\infty} \mathbb{P}(\xi_{(n)} \leqslant x)\mathbb{P}(\eta = n),$$

$$F_{S_{(\eta)}}(x) := \mathbb{P}(S_{(\eta)} \leqslant x) = \sum_{n=0}^{\infty} \mathbb{P}(S_{(n)} \leqslant x)\mathbb{P}(\eta = n).$$

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