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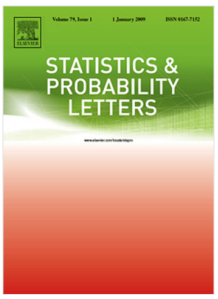

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The joint distribution of the sum and the maximum of heterogeneous exponential random variables

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Abstract

We derive the joint distribution of the sum and the maximum of n independent heterogeneous exponential random variables and provide a detailed description of this new stochastic model for $n = 2$. This generalizes previous results for univariate distributions of the sum and the maximum of heterogeneous exponential random variables as well as their joint distribution in the homogeneous exponential case.

Keywords: Distribution theory; Extremes; Generalized Erlang distribution; Hypoexponential distribution; Order statistics; Peak-to-average ratio

1 Introduction

Motivated by numerous applications, including those in geosciences as well as insurance and finance (see, e.g., Kozubowski et al., 2011 and the references therein), Qeadan et al. (2012) derived the joint distribution of maximum and magnitude of events such as flood, draught, deluge, heat waves, or growth/decline periods of financial or insurance products. In particular, that work included an explicit and exact form of the probability density function (PDF) of the random vector

$$(X, Y) \stackrel{d}{=} \left(\sum_{i=1}^{n} E_i, \bigvee_{i=1}^{n} E_i \right), \quad n \in \mathbb{N} = \{1, 2, ...\}, \tag{1}$$

consisting of the sum X and the maximum Y of n independent and identically distributed (IID) exponential random variables $\{E_i\}$. An extension to the joint distribution of a *random* sum and maximum of N IID exponential random variables (with geometrically distributed N) and trivariate distribution of

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