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On cumulative residual entropy of progressively censored order statistics

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ABSTRACT

The joint cumulative residual entropy (CRE), a measure of information, based on progressively Type-II censored order statistics is discussed. Some useful representations, recurrence relations, a characterization result and two nonparametric estimation methods for the CRE are developed.

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1. Introduction

Let X be an absolutely continuous random variable. Suppose that $X_1, \ldots, X_n$ denote an independently and identically distributed (i.i.d.) random sample of size n taken from a population with cumulative distribution function (cdf) $F(x)$ and probability density function (pdf) $f(x)$, then $X_{1:n} \leq \cdots \leq X_{n:n}$ are the order statistics obtained by arranging the n random variates in an increasing order of magnitude. There are numerous applications of order statistics in different areas such as statistical estimation, reliability analysis, and signal processing (Arnold et al., 1992; David and Nagaraja, 2003).

Information measures of order statistics have been studied extensively in the literature. For example, Park (1995, 2005), Wong and Chen (1990) and Ebrahimi et al. (2004) studied the Shannon entropy of a single order statistic and consecutive order statistics and their spacings. Rao et al. (2004) proposed an alternative notion of entropy called cumulative residual entropy (CRE) for a nonnegative random variable X as $CRE(X) = -\int_{-\infty}^{\infty} \bar{F}(x) \log \bar{F}(x) dx$, where $\bar{F}(x) = 1 - F(x)$ is the survival function (sf) of X. Rao et al. (2004) and Rao (2005) studied the properties of CRE and showed that CRE preserves many important properties of Shannon entropy. Rao (2005) also showed the CRE has some important mathematical properties that can be used in statistical estimation. Recently, Baratpour (2010) discussed the characterization based on CRE of the smallest order statistic and Park and Kim (2014) studied the CRE in the set of the first r order statistics from a sample of size n.

For life-testing experiments in industrial and clinical studies, there are many situations that units (or subjects) are lost or removed from experimentation before the event of interest occurs. The experimenter may not always obtain complete information on the times to event of interest for all experimental units or subjects. Data obtained from such experiments are called censored data. In recent years, progressive Type-II censoring, which is a generalization of the conventional

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Type-II censoring (also known as item-censoring), have received a considerable attention. A progressively Type-II censored experiment can be described as follows: Suppose n identical units are put on a life test and the integer $m < n$ and m nonnegative integers $R_1, \dots, R_m$ such that $R_1 + \cdots + R_m + m = n$ are prefixed, at the time of the first failure, say $X_{1:m:n}$, R_1 of the remaining units are randomly removed. Similarly, at the time of the second failure, $X_{2:m:n}$, R_2 of the remaining units are removed, and so on. Finally, at the time of the mth failure, the rest of the units, $R_m = n - R_1 - \cdots - R_{m-1} - m$, are removed. The observed failures $X_{1:m:n} < \cdots < X_{m:m:n}$ constitute the progressively Type-II censored order statistics (PCOS). The conventional Type-II censoring is a special case of the progressive Type-II censoring when $R_1 = \cdots = R_{m-1} = 0$ and $R_m = n - m$. For further details on progressive censoring, one may refer to Balakrishnan and Aggarwala (2000), Balakrishnan (2007) and Balakrishnan and Cramer (2014).

In this paper, we first derive some important results on CRE of PCOS in Section 2 which are analogue to the results of CRE of usual order statistics in Park and Kim (2014). Specifically, we provide a simple expression of CRE of PCOS with a single integral. Then, in Section 3, we propose a decomposition of CRE in PCOS and use it to derive some useful recurrence relations for the CRE of PCOS. In Section 4, a characterization result based on the CRE is presented. Finally, in Section 5, we discuss two approximation methods to obtain nonparametric estimators of the CRE of PCOS. The proofs of the corollaries, theorems, and recurrence relations are provided in the supplementary material and the singular and plural of an acronym are spelled the same.

2. CRE of progressively Type-II censored order statistics

The joint pdf of the PCOS $X_{1:m:n} < \cdots < X_{m:m:n}$ with censoring scheme $\boldsymbol{R} = (R_1, \dots, R_m)$ can be written as (Balakrishnan and Aggarwala, 2000)

$$f_{1\cdots m:m:n}(\boldsymbol{x}) = f_{1\cdots m:m:n}(x_{1:m:n}, x_{2:m:n}, \dots, x_{m:m:n})$$
$$= c \prod_{i=1}^{m} f(x_{i:m:n}; \theta)[1 - F(x_{i:m:n}; \theta)]^{R_i}, \quad x_{1:m:n} < x_{2:m:n} < \cdots < x_{m:m:n}, \tag{1}$$

where $c = n(n-R_1-1)\cdots(n-R_1-R_2-\cdots-R_{m-1}-m+1)$ and $\boldsymbol{x} = (x_{1:m:n}, \dots, x_{m:m:n})$. The joint sf of $\boldsymbol{X} = (X_{1:m:n}, \dots, X_{m:m:n})$, $\bar{F}_{1\cdots m:m:n}$, can be obtained by integration of the joint pdf in Eq. (1):

$$\bar{F}_{1\cdots m:m:n}(\boldsymbol{x}) = \int_{x_{m:m:n}}^{\infty} \cdots \int_{x_{2:m:n}}^{t_3} \int_{x_{1:m:n}}^{t_2} f_{1\cdots m:m:n}(t_1, t_2, \dots, t_m) dt_1 dt_2 \cdots dt_m,$$
$$x_{1:m:n} < x_{2:m:n} < \cdots < x_{m:m:n}.$$

Then, the joint CRE of the PCOS $\boldsymbol{X} = (X_{1:m:n}, \dots, X_{m:m:n})$ is defined as

$$CRE_{1\dots m:m:n}(\boldsymbol{X})$$
$$= -\int_0^{\infty} \cdots \int_0^{x_{2:m:n}} \bar{F}_{1\cdots m:m:n}(\boldsymbol{x}) \log \bar{F}_{1\cdots m:m:n}(\boldsymbol{x}) dx_{1:m:n} \cdots dx_{m:m:n}. \tag{2}$$

The computation of $CRE_{1\dots m:m:n}(\boldsymbol{X})$ is complicated due to the n-fold integral in Eq. (2) and $\bar{F}_{1\cdots m:m:n}(\boldsymbol{x})$ is depending on the censoring scheme $\boldsymbol{R}$. Abo-Eleneen (2011) provided a simple expression for the entropy of the PCOS, $H_{1\cdots m:m:n}(\boldsymbol{X})$, in terms of the hazard function $h(x) = f(x)/\bar{F}(x)$ as a summation of single integrals $H_{1\cdots m:m:n}(\boldsymbol{X}) = m - \log c - \int_{-\infty}^{\infty} \log h(x) \sum_{i=1}^{m} f_{i:m:n}(x)dx$, where $f_{i:m:n}(x)$ is the pdf of the ith PCOS.

For the conventional Type-II censoring, the CRE of the first m $(m < n)$ order statistics, $\boldsymbol{X}^* = (X_{1:n}, \dots, X_{m:n})$, can be expressed as Park and Kim (2014) $-n \int_0^{\infty} \bar{F}_{m:n-1}(x)\bar{F}(x) \log \bar{F}(x)dx$, where $F_{i:n}(x)$ is the cdf of the ith order statistic based on a sample of size n. Based on the results from Abo-Eleneen (2008, 2011) and Park and Kim (2014), we have the following results.

Lemma 2.1. *Let $X_{1:n}, X_{2:n}, \dots, X_{n:n}$ be the order statistics corresponding to an i.i.d. random sample of size n from a population with pdf $f(x)$, cdf $F(x)$, sf $\bar{F}(x)$ and hazard function $h(x) = f(x)/\bar{F}(x)$. The CRE of $X_{1:n}$ is given by $CRE_{1:n}(X_{1:n}) = -\int_0^{\infty} \frac{f_{1:n}(x)}{h(x)} \log \bar{F}(x)dx$, where $f_{i:n}(x)$ is the pdf of the ith order statistic based on a sample of size n.*

Since $X_{1:m:n} \equiv X_{1:n}$ for any censoring scheme (Ng et al., 2015), therefore, the CRE of the smallest PCOS is

$$CRE_{1:m:n}(X_{1:m:n}) = CRE_{1:n}(X_{1:n}) = -\int_0^{\infty} \frac{f_{1:m:n}(x)}{h(x)} \log \bar{F}(x)dx. \tag{3}$$

In the following theorem, we show that the n-dimension integral in Eq. (2) can be simplified to a single integral.

Theorem 2.1. *For $m < n$, the CRE of PCOS $\boldsymbol{X} = (X_{1:m:n}, X_{2:m:n}, \dots, X_{m:m:n})$ with censoring scheme $\boldsymbol{R} = (R_1, R_2, \dots, R_m)$ can be expressed as*

$$CRE_{1\cdots m:m:n}(\boldsymbol{X}) = -\int_0^{\infty} \frac{1}{h(x)} \log \bar{F}(x) \sum_{i=1}^{m} f_{i:m:n}(x)dx. \tag{4}$$

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