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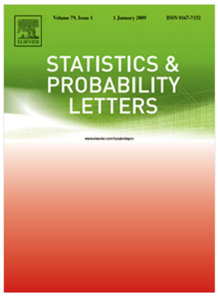

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Approximation of the maximum of storage process with fractional Brownian motion as input*

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Abstract: In this paper, the asymptotic relation between the maximum of the storage process and the maximum of the process sampled at discrete time points is studied. It is shown that these two maxima are asymptotically independent or dependent when the grids of the discrete time points are sufficiently sparse or the so-called Pickands grids. The results complete a gap in Hüsler and Piterbarg (2004) which showed that the two maxima are asymptotically coincident when the grids of the discrete time points are sufficiently dense.

Key Words: extreme values, storage process, fractional Brownian motion, discrete time process

AMS Classification: Primary 60F05; secondary 60G15

1 Introduction

In this paper, we focus on the investigation of storage process, which has the following representation

$$Y(t) = \sup_{s\geq t}(B_H(s) - B_H(t) - c(s-t)), \tag{1}$$

where the constant $c > 0$ is the service rate and $B_H(t), t \geq 0$, is a fractional Brownian motion (fBm) with Hurst parameter $H \in (0, 1)$. The fBm is a centered Gaussian process with stationary increments having almost sure continuous sample paths such that $\mathrm{E}(B_H(t) - B_H(s))^2 = |t-s|^{2H}$, hence with variance $\mathrm{Var}(B_H(t)) = |t|^{2H}$.
This storage process was considered in Piterbarg (2001) who derived results on the large deviations and analysed the supremum $M(T) = \sup_{t\in[0,T]} Y(t)$ of the process $Y(t)$ in a finite interval $[0, T]$. Based on the results of the large deviations, Hüsler and Piterbarg (2004) derived the limit theorem for the supremum $M(T)$, i.e.,

$$P\left\{a_T(M(T) - b_T) \leq x\right\} \to \exp(-e^{-x})$$

as $T \to \infty$, where the constants a_T and b_T will be defined in Section 3. Some further results can be found in Hashorva et al. (2013), Liu et al. (2015), Dębicki and Kosiński (2014,2017) and Dębicki and Liu (2016).
The above limit results can not be used directly, since the available samples are usually over a discrete set of times. Therefore, it is crucial to investigate the asymptotic relation between the maximum $M(T)$ and the maximum of the process $Y(t)$ sampled at discrete time points. Let's define the maximum $M^p(T)$ with respect to $B_H(lp)$ taken on a sufficiently fine discrete grid with mesh $p = p(T) > 0$ as follows:

$$M^p(T) = \sup_{ip\in[0,T]} \widetilde{Y}(ip) \quad \text{with} \quad \widetilde{Y}(ip) = \sup_{l\geq 0}(B_H((l+i)p) - B_H(ip) - clp).$$

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