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On the large deviation principle for maximum likelihood estimator of α-Brownian bridge

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ABSTRACT

We consider the large deviation principle for maximum likelihood estimator of α-Brownian bridge. The full large deviation with explicit rate function is obtained in the case of non-Gaussian limit distribution by local large deviation principle and exponential tightness.

1. Introduction

We consider the following α-Brownian bridge:

$$dX_t = -\frac{\alpha}{T-t}X_t dt + dW_t, \quad X_0 = 0 \tag{1.1}$$

where W_t is a standard Brownian motion, $t \in [0, T)$, $T \in (0, \infty)$ and the parameter $\alpha > 0$ is unknown. The scaling parameter α determines how strong is the force that pulls the process back to 0. In the case $\alpha = 0$, this process is a standard Brownian motion, while in the case $\alpha = 1$, it is the usual Brownian bridge. The α-Brownian bridge is first used to study the arbitrage profit associated with a given futures contract in the absence of transaction costs by Brennan and Schwartz (1990); and has further application in financial theory (see Trede and Wilfling, 2007 and Sondermann et al., 2011).

The maximum likelihood estimator (MLE) of the unknown parameter α is given by (see Barczy and Pap, 2010)

$$\hat{\alpha}_t = -\frac{\int_0^t \frac{X_s}{T-s} dX_s}{\int_0^t \frac{X_s^2}{(T-s)^2} ds}, \quad t \in (0, T).$$

More recently, the α-Brownian bridge has attracted considerable interest in the stochastic community. Barczy and Pap (2011) studied asymptotic behavior of the maximum likelihood estimator $\hat{\alpha}_t$, which is totally different in the following three situations:

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For $\alpha > \frac{1}{2}$,

$$\sqrt{I_\alpha(t)}(\hat{\alpha}_t - \alpha) \xrightarrow{L} \mathcal{N}(0,1) \quad \text{as} \quad t \nearrow T.$$

For $\alpha = \frac{1}{2}$,

$$\sqrt{I_{1/2}(t)}(\hat{\alpha}_t - \frac{1}{2}) \stackrel{L}{=} -\frac{1}{\sqrt{2}} \frac{\int_0^1 W_s dW_s}{\int_0^1 W_s^2 ds}.$$

For $\alpha < \frac{1}{2}$,

$$\sqrt{I_\alpha(t)}(\hat{\alpha}_t - \alpha) \xrightarrow{L} \zeta \quad \text{as} \quad t \nearrow T$$

where $I_\alpha(t) = \int_0^t \frac{\mathbb{E}(X_s^2)}{(T-s)^2} ds$ and ζ is a standard Cauchy distributed random variable. Görgens and Thulin (2014) got the bias-correction for maximum likelihood estimator of α-Brownian bridge; Jiang and Zhao (2011), Görgens (2014) and Zhao and Chen (2017) studied the hypothesis testing problem and asymptotic representation for the tail probability of MLE using large deviation or Karhunen–Lòeve expansions for some α; see also Barczy and Iglói (2011), Es-Sebaiy and Nourdin (2013) and Barczy and Pap (2016).

Using the strategy of change of probability and Gärtner–Ellis theorem, Zhao and Zhou (2013) and Zhao and Liu (2012) studied the large deviation principle (LDP) for the log-likelihood ratio and LDP for MLE of α-Brownian bridge in the case $\alpha > 1/2$ respectively. However, in other cases, the LDP for MLE cannot be obtained by this method since it is not possible to make a direct use of Gärtner–Ellis theorem. Our goal is to establish the large deviation principle in the case $\alpha \le 1/2$ by exponential tightness and local large deviation principle inspired by Bercu and Richou (2017) (see also Bercu and Richou, 2015 and Du Roy de Chaumaray (2016)). We also consider the large deviation principle in the case $\alpha > 1/2$, which gives a more simple proof of the known result in Zhao and Liu (2012).

Our main results are the following:

Theorem 1.1. *Let $\{X_t, t \in [0,T)\}$ be the process given by the SDE (1.1), $\hat{\alpha}_t$ be the maximum likelihood estimator of α.*

(i) *For all $\alpha > 1/2$, $\{\hat{\alpha}_t, t \in [0,T)\}$ satisfies the large deviation principle with speed $\log\left(\frac{T}{T-t}\right)$ and good rate function I defined by*

$$I(x) = \begin{cases} \dfrac{(\alpha - x)^2}{2(2x-1)} & \text{if } x \ge \dfrac{1+\alpha}{3}; \\ \dfrac{2\alpha - 4x + 1}{2} & \text{if } x < \dfrac{1+\alpha}{3}. \end{cases} \tag{1.2}$$

(ii) *For $\alpha = 1/2$, $\{\hat{\alpha}_t, t \in [0,T)\}$ satisfies the large deviation principle with speed $\log\left(\frac{T}{T-t}\right)$ and good rate function I, where*

$$I(x) = \begin{cases} \dfrac{2x-1}{8} & \text{if } x > \dfrac{1}{2}; \\ 1 - 2x & \text{if } x \le \dfrac{1}{2}. \end{cases} \tag{1.3}$$

(iii) *For all $\alpha < 1/2$, $\{\hat{\alpha}_t, t \in [0,T)\}$ satisfies the large deviation principle with speed $\log\left(\frac{T}{T-t}\right)$ and good rate function I, where*

$$I(x) = \begin{cases} \dfrac{(x-\alpha)^2}{2(2x-1)} & \text{if } x > 1-\alpha; \\ \dfrac{1}{2} - \alpha & \text{if } \alpha < x \le 1-\alpha; \\ \alpha - 2x + \dfrac{1}{2} & \text{if } x < \alpha; \\ 0 & \text{if } x = \alpha. \end{cases} \tag{1.4}$$

2. Notions and logarithmic moment generating function

Let us first recall notions of LDP (see Dembo and Zeitouni, 1998). Let $\{\mu_t, t > 0\}$ be a family of probability on a space $(\mathcal{X}, \mathcal{B})$ and $a(t)$ be a nonnegative function such that $\lim_{t\to+\infty} a(t) = +\infty$. A function I from $\mathcal{X}$ to $[0, +\infty]$ is said to be a rate function if it is lower semicontinuous and it is said to be a good rate function if its level set $\{x : I(x) \le a\}$ is compact for all $a \ge 0$. $\{\mu_t, t > 0\}$ is said to satisfy the LDP with speed $a(t)$ and rate function I if for any closed set $F \in \mathcal{B}$,

$$\limsup_{t\to+\infty} \frac{1}{a(t)} \log \mu_t(F) \le -\inf_{x\in F} I(x), \tag{2.1}$$