STAPRO: 8107 Model 3G pp. 1–8 (col. fig: NIL)

ARTICLE IN PRESS

Statistics and Probability Letters xx (xxxx) xxx–xxx

Contents lists available at ScienceDirect

Statistics and Probability Letters

journal homepage: www.elsevier.com/locate/stapro

UNCORRECTED PROOF

Bismut formula for a stochastic heat equation with fractional noise☆

Litan Yan [a,b], Xiuwei Yin [a,*]

[a] College of Information Science and Technology, Donghua University, 2999 North Renmin Rd., Songjiang, Shanghai 201620, PR China
[b] Department of Mathematics, College of Science, Donghua University, 2999 North Renmin Rd., Songjiang, Shanghai 201620, PR China

ARTICLE INFO

Article history:
Received 6 November 2017
Received in revised form 22 November 2017
Accepted 16 January 2018
Available online xxxx

MSC:
60H15
60H35
65C30

Keywords:
Stochastic heat equation
Fractional noise
Bismut formula
Harnack inequality

ABSTRACT

In this note, we establish the Bismut formula for stochastic heat equation

$$\begin{cases} \dfrac{\partial}{\partial t}u(t,x) = \Delta u(t,x) + \dot{W}^H(t,x), \ t \geq 0, \ x \in [0,1], \\ \dfrac{\partial}{\partial x}u(t,x)|_{x=0} = \dfrac{\partial}{\partial x}u(t,x)|_{x=1} = 0, \ t \geq 0, \\ u(0,x) = f(x), \ x \in [0,1], \end{cases}$$

where $f(x) \in \mathbb{H} := L^2([0,1])$ and W^H is the fractional noise with Hurst index $H \in (\frac{1}{2}, 1)$. As an application, we also introduce the Harnack inequality.

© 2018 Elsevier B.V. All rights reserved.

1. Introduction

Since the work of Bismut (1984), the Bismut formula and some related questions for stochastic partial differential equations (SPDEs) have become interesting research aspects in probability. For examples, see Wang and Xu (2012) and Guillin and Wang (2012) for SPDEs via a coupling technique; Dong and Xie (2010) for semi-SPDEs by a simple martingale approach; Bao et al. (2013) and Zhang (2010b) for functional SPDEs using Malliavin calculus method. For more literatures, we refer to Wang (2013), Zhang (2010a, 2013) and the reference.

On the other hand, in recent years, there has been considerable interest in studying fractional Brownian motion due to its some compact properties such as long/short range dependence, self-similarity, stationary increments and Hölder's continuity, and also due to its applications in various scientific areas including telecommunications, turbulence, image processing and finance. It is a suitable generalization of standard Brownian motion. Some surveys and complete literatures could be found in Biagini et al. (2008), Hu (2005), Mishura (2008), Nourdin (2012), Nualart (2006) and Tudor (2013). However, to our best knowledge, there has been little systematic investigation on Bismut formula for SPDEs driven by fractional noise. The main reason for this, in our opinion, is the complexity of dependence structures of solutions to SPDEs. But, for stochastic differential equations driven by fractional Brownian motion. Fan (2013, 2017) studied the related Bismut formulas and applications.

☆ The Project-sponsored by NSFC (11571071), Innovation Program of Shanghai Municipal Education Commission (12ZZ063) and The Fundamental Research Funds for the Central Universities (17D310403).

* Corresponding author.
E-mail addresses: litan-yan@hotmail.com (L. Yan), xweiyin@163.com (X. Yin).

https://doi.org/10.1016/j.spl.2018.01.018
0167-7152/© 2018 Elsevier B.V. All rights reserved.

Please cite this article in press as: Yan L., Yin X., Bismut formula for a stochastic heat equation with fractional noise. Statistics and Probability Letters (2018), https://doi.org/10.1016/j.spl.2018.01.018.

Motivated by these results, in this short note we consider the Bismut formula associated with the following stochastic heat equation with Neumann boundary condition:

$$\begin{cases} \frac{\partial}{\partial t}u(t,x) = \Delta u(t,x) + \dot{W}^H(t,x), \ t \geq 0, \ x \in [0,1], \\ \frac{\partial}{\partial x}u(t,x)|_{x=0} = \frac{\partial}{\partial x}u(t,x)|_{x=1} = 0, \ t \geq 0, \\ u(0,x) = f(x), \ x \in [0,1], \end{cases} \tag{1.1}$$

where $f(x) \in \mathbb{H} := L^2([0,1])$ and W^H is the fractional noise with Hurst index $H > \frac{1}{2}$. Clearly, the solution of the above equation depends with the initial value f. So, we write $u(t,x) = u(t,f,x)$ and $u(t,f) = (t,f,\cdot)$ for all $t \geq 0$. Let $\mathcal{B}_b(\mathbb{H})$ denote the space of all bounded measurable functions on $\mathbb{H}$ and let the operators P_t, $t > 0$ be defined by

$$P_t G(f) = \mathbb{E}[G(u(t,f))]$$

for all $G \in \mathcal{B}_b(\mathbb{H})$. Our objects are to establish the Bismut formula and Harnack inequality for P_t, $t > 0$ under some suitable regularity conditions on f.

The rest of the paper is organized as follows. In Section 2, we recall some basic results about the fractional noise W^H. In Section 3, we prove the main result.

2. Preliminaries

In this section, we briefly recall the definition of the stochastic integration with respect to W^H. For more details, see Biagini et al. (2008), Mishura (2008), Nualart (2006) and the reference.

A centered Gaussian process $W^H = \{W^H(t,A), t \in [0,T], A \in \mathcal{B}([0,1])\}$ defined on a complete probability spaces $(\Omega, \mathcal{F}, (\mathcal{F}_t), \mathbb{P})$ is called a fractional noise if $W^H(0,A) = 0$ for all $A \in \mathcal{B}([0,1])$ and its covariance function admits the representation

$$\mathbb{E}(W^H(t,A)W^H(s,B)) = \frac{1}{2}(t^{2H} + s^{2H} - |t-s|^{2H})\lambda(A \cap B)$$

for all $s, t \in [0,T]$ and $A, B \in \mathcal{B}([0,1])$, where λ is the Lebesgue measure. Throughout this paper we assume that $\frac{1}{2} < H < 1$ and $\alpha_H = H(2H-1)$.

Let $\mathcal{E}$ be the set of step functions on $[0,T] \times [0,1]$ and let $\mathcal{H}$ be the Hilbert space defined as the closure of $\mathcal{E}$ with respect to the scalar product

$$\langle 1_{[0,t]\times A}, 1_{[0,s]\times B}\rangle_{\mathcal{H}} = \mathbb{E}(W^H(t,A)W^H(s,B))$$

for all $s, t \in [0,T]$ and $A, B \in \mathcal{B}([0,1])$. The linear mapping

$$\mathcal{E} \ni \varphi \mapsto W^H(\varphi) := \int_0^T \int_0^1 \varphi(t,x) W^H(dt,dx)$$

defined by $1_{[0,t]\times A} \mapsto W^H(t,A)$ can be extended as an isometry between $\mathcal{H}$ and the Gaussian spaces associated with W^H. This isometry is called the Wiener integral with respect to W^H, denoted by

$$W^H(\varphi) = \int_0^T \int_0^1 \varphi(s,y) W^H(ds,dy)$$

for $\varphi \in \mathcal{H}$.

Consider the kernel function

$$\begin{aligned} K_H(t,s) &= c_H(t-s)^{H-\frac{1}{2}} + c_H(\frac{1}{2} - H)\int_s^t (u-s)^{H-\frac{3}{2}}\left(1 - \left(\frac{s}{u}\right)^{\frac{1}{2}-H}\right)du \\ &= c_H(H - \frac{1}{2})s^{\frac{1}{2}-H}\int_s^t (u-s)^{H-\frac{3}{2}}u^{H-\frac{1}{2}}du \end{aligned}$$

with $t > s > 0$, where $c_H = \left(\frac{2H\Gamma(\frac{3}{2}-H)}{\Gamma(H+\frac{1}{2})\Gamma(2-2H)}\right)^{\frac{1}{2}}$, and define the linear operator K_H^* from $\mathcal{E}$ to $L^2([0,T])$ as follows

$$(K_H^*\varphi)(s,x) = K_H(T,s)\varphi(s,x) + \int_s^T (\varphi(r,x) - \varphi(s,x))\frac{\partial K_H}{\partial r}(r,s)dr.$$

Then, we have

$$(K_H^* 1_{[0,t]\times A})(s,x) = K_H(t,s)1_{[0,t]\times A}$$

Please cite this article in press as: Yan L., Yin X., Bismut formula for a stochastic heat equation with fractional noise. Statistics and Probability Letters (2018), https://doi.org/10.1016/j.spl.2018.01.018.

UNCORRECTED PROOF

Download English Version:

https://daneshyari.com/en/article/7548347

Download Persian Version:

https://daneshyari.com/article/7548347

Daneshyari.com