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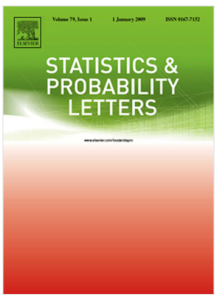

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Ryoichi Suzuki*

Abstract

In this paper, we consider Malliavin differentiability of indicator functions on canonical Lévy spaces. We give necessary and sufficient conditions for it. This is a Lévy space version of a result of Sekiguchi and Shiota [21].

Keywords: Malliavin calculus; Lévy processes; indicator functions; digital options
2010 Mathematics Subject Classification. Primary 60H07; Secondary 60G51.

1 Introduction

The Malliavin calculus (stochastic calculus of variations) is an infinite-dimensional differential calculus on the Wiener space (Lévy spaces), which was first introduced by Paul Malliavin ([14]). It has many applications in probability theory, stochastic analysis, partial differential equations, mathematical finance and many others (see, e.g., the following books [5, 7, 12, 15, 16, 19, 20]). In the theory of Malliavin calculus, Malliavin differentiability of random variables is one of the most important problems because Malliavin derivative leads to many useful results. On the other hand, the indicator function $\mathbf{1}_A(x)$ of the set A is the function

$$\mathbf{1}_A(x) = \begin{cases} 1 & \text{for } x \in A, \\ 0 & \text{for } x \notin A. \end{cases}$$

Indicator functions appear everywhere in mathematics and related fields. Especially, they are often used in probability theory, statistics and related fields to simplify notation and to prove theorems. Hence, the problem for Malliavin differentiability of indicator function is important. In this paper, we consider Malliavin differentiability of indicator function on canonical Lévy space.

On the Wiener space $(\Omega_W, \mathcal{F}_W, \mathbb{P}_W)$, we have already known the following:

Proposition 1.1 ([21]) *If $A \in \mathcal{F}_W$, then, the indicator function of A belongs to $\mathbb{D}_W^{1,2}$ if and only if $\mathbb{P}_W(A)$ is equal to zero or one, where $\mathbb{D}_W^{1,2}$ is the Sobolev space on the Wiener space.*

See also Proposition 1.15 in [16] and Proposition 1.26 in [15]. We show a canonical Lévy space version of Proposition 1.1 in this article. Di Nunno et al. ([10]) and Okur [18] introduced an example of indicator function (digital option) which dosen't belong to $\mathbb{D}^{1,2}$, where $\mathbb{D}^{1,2}$ is stochastic Sobolev space for functionals of Lévy processes. However, they did not give necessary and sufficient conditions for Malliavin differentiability of indicator functions. Hence, we give them. Moreover, we have already known the following:

Proposition 1.2 (Proposition 1.16 in [16]) *Let $F \in \mathbb{D}_W^{1,2}$ be such that*

$$\mathbb{E}\left[|F|^{-2}\int_0^T |D_t^W F|^2 dt\right] < \infty,$$

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