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On information-based residual lifetime in survival models with delayed failures

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ABSTRACT

At many practical instances, the initiating point events (e.g., shocks) affect an object not immediately, but after some random delay. These models were studied in the literature only for the case when an initial shock process is Poisson. In our paper, we generalize these results to a meaningful case of the generalized Polya process (GPP) of initial shocks that was recently introduced in the literature. Distinct from the Poisson process, the GPP possesses the property of dependent increments, which makes it much more attractive in applications. We derive the distribution of the time to failure for a system subject to the GPP with delays. Our main focus, however, is on analysis of the corresponding residual lifetime distribution that depends now on the full history (information) of the initiating shock process.

1. Introduction

Let $\{N(t), t \geq 0\}$ be an orderly point process (without multiple occurrences) of some 'initiating' events (IEs) with arrival times $T_1 < T_2 < T_3 < \cdots$. Assume that each event from this process triggers the 'effective event' (EE), which occurs after a random time (delay) $D_i, i = 1, 2, \ldots$, since the occurrence of the corresponding IE at T_i (Cha and Finkelstein, 2012). This setting can be encountered in many practical situations, when, e.g., initiating events start the process of developing the non-fatal faults in a system and we are interested in the number of these faults in $[0, t)$. For instance, the initiation of cracks in a material was modeled in Castro et al. (2015) by the nonhomogeneous Poisson process (NHPP), where $D_i, i = 1, 2, \ldots$ were assumed to be i.i.d random variables (see also Kuniewski et al., 2009, for the NHPP case). In insurance loss models, claims or losses are reported with some delays from the events causing them (see, e.g., Guo et al., 2013).

Alternatively, each EE can result in a fatal, terminating failure and then one can be interested in the survival probability of a system. This means that the first EE results in the failure of our system. Note that the IEs can often be interpreted as some external shocks affecting a system, and, for convenience, we will use this term (interchangeably with the "IE"). Various shock models (without delays) were intensively studied in the literature (e.g., Al-Hameed and Proschan, 1973; Esary et al., 1973; Lemoine and Wenocur, 1986; Cha and Mi, 2007; Nakagawa, 2007; Finkelstein, 2008; Frostig and Kenzin, 2009; Cha and Finkelstein, 2009, 2011; Montoro-Cazorla and Pérez-Ocón, 2011; Huynh et al., 2012; Finkelstein and Cha, 2013).

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Cha and Finkelstein (2012) have provided a thorough probabilistic analysis of the ‘model with delays’ when initiating events follow the NHPP. In the current paper, we present a meaningful generalization of these results to the case when initiating events follow the generalized Polya process (GPP), which was recently introduced and characterized in Cha (2014). Specifically, it was proved in this paper that the GPP possesses the positive dependent property, which can be crucial in various real applications, where the assumption of independent increments of the NHPP was often used just for simplicity.

Kuniewski et al. (2009) showed that when the process of initiating events is the NHPP, the process of effective events is also NHPP. It turns out that this is a rather unique property. Indeed, denote the rate of the IEs of a general ordinary process by $r(t)$. Then the rate of the point process with delays (EEs) is

$$r_D(t) = \int_0^t r(x)g(t-x)dx, \tag{1}$$

where ‘D’ stands for “delay” and $g(t)(G(t))$ is the pdf (Cdf) of the i.i.d. D_i, $i = 1, 2, \ldots$. For instance, for the HPP with $r(t) = \lambda$, the rate of the NHPP of EEs, $r_D(t) = \lambda G(t)$ just follows the shape of the distribution of the delay increasing asymptotically to λ. When the process of IEs is renewal and $g(t)$ is decreasing (e.g., for exponential distribution), we can apply the key renewal theorem (Ross (1996)) as $t \to \infty$,

$$r_D(t) = \frac{1}{\mu}(1 + o(1)),$$

where μ is the mean of the inter-arrival time for the renewal process. Thus the rate of the corresponding EE process is asymptotically tending to a constant; however, this process is not a renewal one any more, as the EEs are not the renewal points for the process now. Hence, the type of the initial process is preserved for the process with delays in the NHPP case, whereas it is not preserved even for the renewal process.

Our main focus in this paper is in the *novel* residual lifetime concept for the failure model in the GPP process. The residual lifetime plays an important role in reliability, survival analysis, demography and other disciplines. However, in a classical context, it is defined *only* by the distribution to failure and the current time. In our case, with IEs modeled by the GPP process of shocks, it *differs dramatically* from the classical notion due to dependence on history (current information) of the IE shock process. In fact, it is a full history of the process (not only the number of shocks but the arrival times as well).

The paper is organized as follows. In Section 2, we briefly introduce the GPP and we derive the survival function and the failure rate function of the system. In Section 3, we discuss and characterize the new notion of the history-based residual lifetime. Finally, concluding remarks are given in Section 4.

2. A shock model with delayed termination

A new counting process called the ‘Generalized Polya Process’ (GPP) has been recently introduced and studied in Cha (2014), where it is characterized via the corresponding stochastic intensity. The stochastic intensity for an orderly point process $\{N(t),\ t \geq 0\}$, is defined as the following limit (see also Aven and Jensen, 1999, 2000; Cha and Finkelstein, 2011):

$$\lambda_t = \lim_{\Delta t \to 0} \frac{\Pr[N(t, t+\Delta t) = 1 | H_{t-}]}{\Delta t} = \lim_{\Delta t \to 0} \frac{E[N(t, t+\Delta t) | H_{t-}]}{\Delta t}, \tag{2}$$

where $N(t_1, t_2)$, $t_1 < t_2$, represents the number of events in $[t_1, t_2)$ and $H_{t-} \equiv \{N(u), 0 \leq u < t\}$ is the history (internal filtration) of the process in $[0, t)$. A formal definition of the GPP (Cha, 2014) is given as

Definition 1 (*Generalized Polya Process (GPP)*). A counting process $\{N(t),\ t \geq 0\}$ is called the Generalized Polya Process (GPP) with the set of parameters $(\lambda(t), \alpha, \beta)$, $\alpha \geq 0$, $\beta > 0$, if

(i) $N(0) = 0$;
(ii) $\lambda_t = (\alpha N(t-) + \beta)\lambda(t)$.

Note that the GPP with $(\lambda(t), \alpha = 0, \beta = 1)$ corresponds to the NHPP with the rate $\lambda(t)$. Thus, the GPP is a generalization of the NHPP.

We now generalize the results of Cha and Finkelstein (2012) obtained for the NHPP of initiating events to the case of the GPP. This is a meaningful and not a straightforward generalization as the suggested model can account now for the *history* (previous shocks), which is important in practice. The assumption of the NHPP was made previously just for simplicity and now we are able to relax it.

Consider a system subject to the GPP (with the set of parameters $(\lambda(t), \alpha, \beta)$) of IEs $\{N(t), t \geq 0\}$, to be called, for convenience, shocks. Let the corresponding arrival times be denoted as $T_1 < T_2 < T_3 \ldots$. The sequence of EEs $\{T_i + D_i\}$, $i = 1, 2, \ldots$, after ordering in the ascending order, form a new point process, $\{N_E(t),\ t \geq 0\}$, where D_i, $i = 1, 2, ..$ are i.i.d., non-negative random variables with the pdf (Cdf) $g(t)(G(t))$. We assume that the sequence $\{D_i, i = 1, 2, \ldots\}$ and the shock process $\{N(t), t \geq 0\}$ are independent. Denote also the time to the first event in this process, which is the survival time if the EEs are fatal, by T_S. Thus, in this case T_S is considered as the time to failure of our system. The following result provides the survival function and the failure rate of T_S.