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Calculation of sound radiant efficiency and sound radiant modes of arbitrary shape structures by BEM and general eigenvalue decomposition

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Abstract

In this paper, a numerical method is presented to calculate sound radiant efficiency and radiant modes of arbitrary shape structures. Some methods have been proposed to compute sound radiant efficiencies and sound radiant modes of plates and beams. However, there is not a valid method to calculate for arbitrary shape structures except for measurement at the present time. The method proposed can predicate the sound radiant efficiencies and the sound radiant modes for arbitrary shape structures by boundary element method (BEM) and general eigenvalue decomposition. The validity of this method is demonstrated by two numerical examples of pulsating sphere and radiation cube.

Keywords: Sound radiation; Sound radiant mode; Boundary element method; Sound radiant efficiency

1. Introduction

The prediction and control of acoustic radiation become a more and more important issue in new product design process. The sound radiant efficiency and sound power are important parameters to describe the radiant characteristic of structures. However, it is well known that the total sound power can not be directly calculated by the modal efficiencies when radiant efficiencies are described by structural modes shape function. The main reason is that the radiation of structures modes is strongly coupled each other, which is also the main difficulty of controlling and computing sound radiation. Many scholars have studied the sound radiation behavior and tried to find the connection between the sound radiation and vibration, and to find an approach to decouple the sound radiation.

Photiadis [1], Cunefare et al. [2,3], Currey and Cunefare [4] and Elliott [5] presented the concept of the acoustical radiant modes during early 1990s. The advantage of this approach is the elimination of the complexity of the structure mode's coupling terms. The sound power of structures can be expanded as the sum of sound radiant modes. For plate and beam, Photiadis [1], Cunefare et al. [2,3], Currey and Cunefare [4] have studied their radiant modes and radiant efficiencies using the concept of the acoustical radiant modes. Snyder and Tanaka [6] clarified the coupling reason of sound radiation by structures modes calculation. As an example, he analyzed the radiation characteristic of radiant modes of thin plates. Oppenheimer and Dubowsky [7] tested the radiant efficiency of plates by experiment. Elliott [5] and Koorosh et al. [8,9] applied this idea to study the active control of the sound radiation. The results showed that this method could obtain notable effect.

However, for complex structures, such as arbitrary enclosing shape structures, there is not a valid method to compute the sound radiant efficiency and sound radiant

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modes except for measurement. Hashimoto [10] have presented a method to measure sound radiant efficiency.

In this paper, a numerical approach is proposed to calculate the sound radiant efficiency and sound radiant modes of arbitrary shape structures by boundary element method and general eigenvalue decomposition.

Boundary element method (BEM) is a validity method to compute structure-born acoustical radiation of complex structures [11–15]. In this paper, the surface pressure of structures is computed via BEM, and then the sound power can be expressed as a Hermitian quadratic form, and the equivalent sound power can be expressed as a quadratic form. As the impedance matrix of sound power of a structures is positive definite, and the coupling matrix of mean square of the equivalent sound power is real symmetrical and positive definite, the sound radiation can be decoupled via general eigenvalue decomposition, the radiant modes of structures are the eigenvectors of eigenvalues. The sound radiant efficiencies and sound radiant power can be expanded as the sum of the sound radiant modes if radiant modes are taken as a basis.

2. Basic boundary element formulation

Consider the acoustic pressure field in the exterior unbound domain. The governing differential equation of the exterior acoustic domain in steady-state linear acoustics is the classical Helmholtz equation [11] as follows

$$\nabla^2 p + k^2 p = 0 \tag{1}$$

where p is the sound pressure, $k = \omega/c$ is the wavenumber, ω and c are the angular frequency and speed of sound, respectively.

The Neumann boundary condition and the Sommerfeld radiation condition at infinity can be expressed as

$$\frac{\partial p}{\partial n} = -\mathrm{i}\rho\omega v_n \tag{2}$$

$$\lim_{r\to\infty}\left[r\left(\frac{\partial p}{\partial r} - \mathrm{i}kp\right)\right] = 0 \tag{3}$$

where $\mathrm{i} = \sqrt{-1}$, and ρ is the density of the fluid.

For arbitrary enclosed structures, including those with corners and edges, the form of Helmholtz integral equation [11,13] is given by

$$C(P)p(P) = \int_S \left(p\frac{\partial G}{\partial n} + \mathrm{i}\rho\omega v_n G\right)\mathrm{d}S \tag{4}$$

where n is the unit normal vector pointing into the acoustic domain, and G is the fundamental solution of the inhomogeneous Helmholtz equation, namely the Green's function of free space, and $\partial G/\partial n$ is derivative of G with respect to n. They can be written as follows:

$$\begin{cases} G(r) = \frac{e^{-\mathrm{i}kr}}{r} \\ \frac{\partial G}{\partial n} = -\left(\mathrm{i}k + \frac{1}{r}\right)G\frac{\partial r}{\partial n} \end{cases} \tag{5}$$

where $r = \mid \bar{x}_P - \bar{x}_Q \mid$, and x_Q is any point in space. For any point P in space, the value of coefficient $C(P)$ can be expressed as [13]

$$C(P) = \begin{cases} 1 & P \in V \\ 4\pi + \int_S \frac{\partial}{\partial n}\left(\frac{1}{r}\right)\mathrm{d}S & P \in S \\ 0 & P \in (V \cup S) \end{cases} \tag{6}$$

The evaluation of Eq. (4) was performed using isoparametric element and numerical quadrature. For isoparametric element, interpolation of the pressure and velocity at the nodes determines the pressure and velocity distributions over entire element [16]

$$\begin{cases} p = \sum_{l=1}^{4} N_l p_l \\ v = \sum_{l=1}^{4} N_l v_l \end{cases} \tag{7}$$

where $p_l = (p_1, p_2, p_3, p_4)^{\mathrm{T}}$, $v_l = (v_1, v_2, v_3, v_4)^{\mathrm{T}}$,$p_l$ and v_l represent the pressure and velocity of nodes of element, respectively, and N_l is an interpolation shape function

$$N_l = \frac{1}{4}(1 + \xi_l\xi)(1 + \eta_l\eta) \quad l = 1, 2, 3, 4 \tag{8.1}$$

The exactly formula of the shape function is

$$N_1 = \frac{1}{4}(1 + \xi)(1 + \eta) \tag{8.2}$$

$$N_2 = \frac{1}{4}(1 - \xi)(1 + \eta) \tag{8.3}$$

$$N_3 = \frac{1}{4}(1 - \xi)(1 - \eta) \tag{8.4}$$

$$N_4 = \frac{1}{4}(1 + \xi)(1 - \eta) \tag{8.5}$$

where ξ and η is local coordinate.

The numerical evaluation of Eq. (4) for the nodes of each element ultimately yields a system of algebraic equation [16]

$$H_H P = G_H V \tag{9}$$

so

$$P = (H_H^{-1} G_H)V = ZV \tag{10}$$

where $P = (p_1, p_2, \ldots, p_{\mathrm{n}})^{\mathrm{T}}$, $V = (v_1, v_2, \ldots, v_{\mathrm{n}})^{\mathrm{T}}$, $Z = H_H^{-1}G_H$.

It is well known that the classical boundary element method for exterior acoustical problem fails to provide a unique solution at certain frequency. Several modified integral formulations have been developed to overcome the problem. By far, the CHIEF method [11,15] is the most popular method, and it is adopted in this study.

For exterior sound radiation problem, if one chooses some CHIEF points out of domain V and S, these points, according to Eq. (6), satisfy the Helmholtz equation [11]

$$\int_S \left(p\frac{\partial G}{\partial n} + \mathrm{i}\rho\omega v_n G\right)\mathrm{d}S = 0 \tag{11}$$

Introduce Eq. (7) into Eq. (11), one may get