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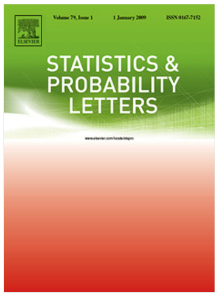

Tail bounds for sums of geometric and exponential variables

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TAIL BOUNDS FOR SUMS OF GEOMETRIC AND EXPONENTIAL VARIABLES

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ABSTRACT. We give explicit bounds for the tail probabilities for sums of independent geometric or exponential variables, possibly with different parameters.

1. Introduction and notation

Let $X = \sum_{i=1}^n X_i$, where $n \geqslant 1$ and X_i, $i = 1, \dots, n$, are independent geometric random variables with possibly different distributions: $X_i \sim \mathrm{Ge}(p_i)$ with $0 < p_i \leqslant 1$, i.e.,

$$\mathbb{P}(X_i = k) = p_i(1-p_i)^{k-1}, \qquad k = 1, 2, \dots. \tag{1.1}$$

Our goal is to estimate the tail probabilities $\mathbb{P}(X \geqslant x)$. (Since X is integer-valued, it suffices to consider integer x. However, it is convenient to allow arbitrary real x, and we do so.)

We define

$$\mu := \mathbb{E}\, X = \sum_{i=1}^n \mathbb{E}\, X_i = \sum_{i=1}^n \frac{1}{p_i}, \tag{1.2}$$

$$p_* := \min_i p_i. \tag{1.3}$$

We shall see that p_* plays an important role in our estimates, which roughly speaking show that the tail probabilities of X decrease at about the same rate as the tail probabilities of $\mathrm{Ge}(p_*)$, i.e., as for the variable X_i with smallest p_i and thus fattest tail.

Recall the simple and well-known fact that (1.1) implies that, for any non-zero z such that $|z|(1-p_i) < 1$,

$$\mathbb{E}\, z^{X_i} = \sum_{k=1}^\infty z^k\, \mathbb{P}(X_i = k) = \frac{p_i z}{1-(1-p_i)z} = \frac{p_i}{z^{-1} - 1 + p_i}. \tag{1.4}$$

For future use, note that since $x \mapsto -\ln(1-x)$ is convex on $(0,1)$ and 0 for $x = 0$,

$$-\ln(1-x) \leqslant -\frac{x}{y}\ln(1-y), \qquad 0 < x \leqslant y < 1. \tag{1.5}$$

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Key words and phrases. geometric distribution; exponential distribution; tail bounds.

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