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TAIL BOUNDS FOR SUMS OF GEOMETRIC AND EXPONENTIAL VARIABLES

SVANTE JANSON

ABSTRACT. We give explicit bounds for the tail probabilities for sums of independent geometric or exponential variables, possibly with different parameters.

1. INTRODUCTION AND NOTATION

Let $X = \sum_{i=1}^n X_i$, where $n \geq 1$ and X_i , $i = 1, \dots, n$, are independent geometric random variables with possibly different distributions: $X_i \sim \text{Ge}(p_i)$ with $0 < p_i \leq 1$, i.e.,

$$\mathbb{P}(X_i = k) = p_i(1 - p_i)^{k-1}, \quad k = 1, 2, \dots \quad (1.1)$$

Our goal is to estimate the tail probabilities $\mathbb{P}(X \geq x)$. (Since X is integer-valued, it suffices to consider integer x . However, it is convenient to allow arbitrary real x , and we do so.)

We define

$$\mu := \mathbb{E} X = \sum_{i=1}^n \mathbb{E} X_i = \sum_{i=1}^n \frac{1}{p_i}, \quad (1.2)$$

$$p_* := \min_i p_i. \quad (1.3)$$

We shall see that p_* plays an important role in our estimates, which roughly speaking show that the tail probabilities of X decrease at about the same rate as the tail probabilities of $\text{Ge}(p_*)$, i.e., as for the variable X_i with smallest p_i and thus fattest tail.

Recall the simple and well-known fact that (1.1) implies that, for any non-zero z such that $|z|(1 - p_i) < 1$,

$$\mathbb{E} z^{X_i} = \sum_{k=1}^{\infty} z^k \mathbb{P}(X_i = k) = \frac{p_i z}{1 - (1 - p_i)z} = \frac{p_i}{z^{-1} - 1 + p_i}. \quad (1.4)$$

For future use, note that since $x \mapsto -\ln(1 - x)$ is convex on $(0, 1)$ and 0 for $x = 0$,

$$-\ln(1 - x) \leq -\frac{x}{y} \ln(1 - y), \quad 0 < x \leq y < 1. \quad (1.5)$$

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