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Strong local nondeterminism of spherical fractional Brownian motion

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ABSTRACT

Let $B = \left\{B(x),\ x \in \mathbb{S}^2\right\}$ be the fractional Brownian motion indexed by the unit sphere $\mathbb{S}^2$ with index $0 < H \le \frac{1}{2}$, introduced by Istas 2015. We establish optimal upper and lower bounds for its angular power spectrum $\{d_\ell, \ell = 0, 1, 2, \ldots\}$, and then exploit its high-frequency behavior to establish the property of its strong local nondeterminism of B.

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1. Introduction

The spherical fractional Brownian motion (SFBM, for brevity) was introduced by Istas in 2005 (Istas, 2005), as an extension of the spherical Brownian motion of Lévy (1965) as well as a spherical analogue of fractional Brownian motion indexed by the Euclidean spaces. Later Istas (2006, 2007) established the Karhunen–Loève expansion and studied quadratic variations of the spherical fractional Brownian motion.

The purpose of this paper is to investigate the property of strong local nondeterminism (SLND) for the spherical fractional Brownian motion. This is motivated by studies of sample path properties of Gaussian random fields indexed by the Euclidean space $\mathbb{R}^N$ and by the currently increasing interest in stochastic modeling of spherical data in statistics, cosmology and other applied areas (see below).

The concept of local nondeterminism (LND) of a Gaussian process was first introduced by Berman (1973) to unify and extend his methods for studying the existence and joint continuity of local times of real-valued Gaussian processes. Roughly speaking, a Gaussian process is said to have the LND property if it has locally approximately independent increments, see Berman (1973, Lemma 2.3) for precise description. This property allowed Berman to overcome some difficulties caused by the complex dependence structure of a non-Markovian Gaussian process for studying its local times. Pitt (1978) and Cuzick (1982) extended Berman's LND to Gaussian random fields. However, the property of LND is not enough for establishing fine regularity properties such as the law of the iterated logarithm and the uniform modulus of continuity for the local times or self-intersection local times of Gaussian random fields. For studying these and many other problems on Gaussian

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random fields, the appropriate properties of strong local nondeterminism (SLND) have proven to be more powerful. Instead of recalling definitions of various forms of strong local nondeterminism for (isotropic or anisotropic) Gaussian random fields indexed by $\mathbb{R}^N$ and their applications, we refer to Xiao (2007, 2009, 2013) for more information.

Recently, Lan et al. (2016) have studied the SLND property of a class of Gaussian random fields indexed by the unit sphere $\mathbb{S}^2$, which are also called spherical Gaussian random fields. The main difference between (Lan et al., 2016) and the aforementioned work for Gaussian fields indexed by the Euclidean space is that Lan et al. (2016) takes the spherical geometry of $\mathbb{S}^2$ into full consideration and its method relies on harmonic analysis on the sphere. More specifically, Lan, Marinucci and Xiao have considered a centered isotropic Gaussian random field $T = \{T(x), x \in \mathbb{S}^2\}$. That is, T satisfies

$$\mathbb{E}\big(T(x)T(y)\big) = \mathbb{E}\big(T(gx)T(gy)\big) \tag{1}$$

for all $g \in SO(3)$ which is the group of rotations in $\mathbb{R}^3$. See Marinucci and Peccati (2011) for a systematic account on random fields on $\mathbb{S}^2$. By applying harmonic analytic tools on the sphere, Lan et al. (2016) have proved that the SLND property of an isotropic Gaussian field T on $\mathbb{S}^2$ is determined by the high-frequency behavior of its angular power spectrum. Moreover, by applying SLND, they have established exact uniform modulus of continuity for a class of isotropic Gaussian fields on $\mathbb{S}^2$.

Since SFBM $B = \{B(x), x \in \mathbb{S}^2\}$ is not isotropic in the sense of (1), the results on SLND in Lan et al. (2016) are not directly applicable. In our approach we will make use of the Karhunen–Loève expansion of the spherical fractional Brownian motion obtained by Istas (2006) and derive optimal upper and lower bounds for the coefficients $\{d_\ell, \ell = 0, 1, \ldots\}$ (see (7) for the definition). These bounds for $\{d_\ell\}$ correct the last part of Theorem 1 in Istas (2006) and will be useful for studying the dependence structures and sample path properties of SFBM. This paper provides an important step towards this direction. More specifically, we demonstrate that the coefficients $\{d_\ell\}$ play the same role as the angular power spectrum of an isotropic Gaussian field on $\mathbb{S}^2$ in Lan et al. (2016) and their high frequency behavior determines the property of strong local nondeterminism of SFBM. For this reason, we will also call the sequence $\{d_\ell, \ell = 0, 1, \ldots\}$ the angular power spectrum of SFBM.

Similarly to the cases of Gaussian random fields indexed by the Euclidean space $\mathbb{R}^N$ (cf. Xiao, 2007, 2009, 2013), we expect that the SLND property in Theorem 3.2 will be useful for studying regularity (e.g., the exact modulus of continuity, exact modulus of nondifferentiability, etc.) and fractal properties of SFBM. This will be carried out in a subsequent paper (Lan and Xiao, 2017).

Our analysis on SFBM and other spherical Gaussian random fields is strongly motivated by applications in a number of scientific areas, such as geophysics, astrophysics, cosmology, and atmospheric sciences (see e.g. Alkhaled et al., 2008, Cabella and Marinucci, 2009, de Boyer Montégut et al., 2004, Dodelson, 2003). Huge data sets from satellite missions such as the Wilkinson Microwave Anisotropy Probe (*WMAP*) of NASA (see http://map.gsfc.nasa.gov/) and the *Planck* mission of the European Space Agency (see http://sci.esa.int/planck/53103-planck-cosmology/) have been collected and made publicly available. Spherical random fields (usually assumed to be Gaussian) have been proposed for modeling such data sets.

Related to aforementioned aspects, we also mention that in probability and statistics literature various isotropic or anisotropic Gaussian random fields on $\mathbb{S}^2$ have been constructed and studied (see e.g. Estrade and Istas, 2010, Huang et al., 2011, Huang et al., 2012, Jun and Stein, 2008, Panchenko and Talagrand, 2007). Excursion probabilities and topological properties of excursion sets of isotropic Gaussian random fields on $\mathbb{S}^2$ have been studied in Cheng and Xiao (2016) and Marinucci and Vadlamani (2016). Many interesting questions on probabilistic and statistical properties of anisotropic Gaussian random fields on the sphere can be raised. In order to study these problems, it would be interesting to establish appropriate properties of strong local nondeterminism for anisotropic Gaussian random fields on $\mathbb{S}^2$.

The rest of the paper is organized as follows. In Section 2, we recall briefly some background material on SFBM, including its Karhunen–Loève expansion from Istas (2006), and an analysis of its random coefficients. Our main result in this section is Theorem 2.2, which provides optimal upper and lower bounds for the angular power spectrum $\{d_\ell, \ell = 0, 1, \ldots\}$ of SFBM. In Section 3, we combine the high frequency behavior of $\{d_\ell\}$ and Proposition 7 in Lan et al. (2016) to establish the property of strong local nondeterminism for SFBM.

2. SFBM and asymptotic behavior of its angular power spectrum

Let N be the North pole on $\mathbb{S}^2$, and $d_{\mathbb{S}^2}(x, y)$ the geodesic distance between x and y on $\mathbb{S}^2$. Recall from Istas (2005) the definition of SFBM.

Definition 2.1. The SFBM $B = \{B(x), x \in \mathbb{S}^2\}$ is a centered Gaussian random field with

$$B(N) = 0, \ a.s. \tag{2}$$

and

$$\mathbb{E}[B(x) - B(y)]^2 = d_{\mathbb{S}^2}(x, y)^{2H}, \tag{3}$$

for any $x, y \in \mathbb{S}^2$ and $0 < H \le \frac{1}{2}$.

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