Statistics and Probability Letters 135 (2018) 92–101

Contents lists available at ScienceDirect

Statistics and Probability Letters

journal homepage: www.elsevier.com/locate/stapro

American options under periodic exercise opportunities

José-Luis Pérez [a,*], Kazutoshi Yamazaki [b]

[a] Department of Probability and Statistics, Centro de Investigación en Matemáticas, A.C., Calle Jalisco s/n, C.P. 36240, Guanajuato, Mexico

[b] Department of Mathematics, Faculty of Engineering Science, Kansai University, 3-3-35 Yamate-cho, Suita-shi, Osaka, 564-8680, Japan

ARTICLE INFO

Article history:
Received 17 August 2017
Received in revised form 26 October 2017
Accepted 29 November 2017
Available online 15 December 2017

MSC:
60G40
60J75
91G80

Keywords:
American options
Optimal stopping
Lévy processes
Periodic exercise opportunities

ABSTRACT

In this paper, we study a version of the perpetual American call/put option where exercise opportunities arrive only periodically. Focusing on the exponential Lévy models with i.i.d. exponentially-distributed exercise intervals, we show the optimality of a barrier strategy that exercises at the first exercise opportunity at which the asset price is above/below a given barrier. Explicit solutions are obtained for the cases where the underlying Lévy process has only one-sided jumps.

1. Introduction

The valuation of the perpetual American put/call options has been considered in many papers. This can be used as an approximation to the finite maturity case, and also used in various real option models (see, e.g., Dixit and Pindyck, 1994). Unlike the finite maturity case that commonly requires numerical approaches, the perpetual case typically admits a simple analytical solution. In particular, when the underlying process is an exponential Lévy process, the optimal stopping time is known to be of barrier-type: it is optimal to exercise as soon as the process goes above or below a certain barrier, which can be written concisely using the Wiener–Hopf factors (see Remark 2.1). We refer the readers to the seminal papers by Mordecki (2002) and Alili and Kyprianou (2005), in this context.

In this paper, we consider a variant where exercise opportunities arrive only periodically. While most of the continuous-time models assume that one can exercise the option instantaneously at any time, in reality one can monitor the underlying process only at intervals. Motivated by this, we consider the case in which one can exercise only at the jump times of an independent Poisson process. This can be seen as a modification of Bermudan options where the constant exercise intervals are replaced by i.i.d. exponential random variables.

We consider both put- and call-type payoffs. The objective of this paper is twofold.

First, for a general underlying Lévy process, we show the optimality of the *periodic barrier strategy* that exercises at the first exercise opportunity at which the underlying process is below/above a suitably chosen barrier.

Second, we focus on the spectrally one-sided Lévy case (where jumps are one-sided) and obtain the optimal solution explicitly using the scale function. The expected value under a periodic barrier strategy can be directly computed using the results in Albrecher et al. (2014). We obtain the optimal barriers and derive explicit forms of the optimal value function.

* Corresponding author.
E-mail addresses: jluis.garmendia@cimat.mx (J.-L. Pérez), kyamazak@kansai-u.ac.jp (K. Yamazaki).

https://doi.org/10.1016/j.spl.2017.11.020

In order to confirm the analytical results, we also give numerical results using spectrally one-sided Lévy processes consisting of a Brownian motion with i.i.d. exponential-size jumps. We confirm the optimality, and also study the behavior of the optimal solutions with respect to the rate of Poisson arrivals.

This paper is motivated by recent developments on the optimal dividend problem where one wants to maximize the total discounted dividends until ruin, with an extra restriction that the dividend payment opportunities arrive only periodically. It has recently been shown, for the case of exponential interarrival times, that a periodic barrier strategy is optimal when the underlying process is a spectrally one-sided Lévy process (see Avanzi et al., 2014; Noba et al., 2017; Pérez and Yamazaki, 2017). This current paper can be seen as its optimal stopping version. Other related research includes Parisian ruin/reflection (see, e.g., Avram et al., forthcoming; Pérez and Yamazaki, 2016); when the (Parisian) delays are exponential random variables, many fluctuation identities can be written semi-analytically in terms of the scale function, similarly to what we discuss in this paper for the case of a spectrally negative Lévy process.

The rest of the paper is organized as follows. Section 2 gives a mathematical formulation of the problem. Section 3 shows, for a general Lévy case, the optimality of a periodic barrier strategy. We then obtain optimal solutions explicitly for the spectrally one-sided case in Section 4. We conclude with numerical results in Section 5.

2. Our problem

Let X be a Lévy process and $S = \exp X$ the price of a stock. For $s > 0$, we denote by $\mathbb{P}_s$ the law of S when it starts at s ($X_0 = \log s$) and write for convenience $\mathbb{P}$ in place of $\mathbb{P}_1$. Accordingly, we shall write $\mathbb{E}_s$ and $\mathbb{E}$ for the associated expectation operators. We define $\mathcal{T} := \{T_1, T_2, \dots\}$ as the jump times of an independent Poisson process N with rate $\lambda > 0$. Let $\mathbb{F}$ be the filtration generated by (X, N) and $\mathbb{T}$ the set of $\mathbb{F}$-stopping times. The set of strategies is given by $\mathcal{T} \cup \{\infty\}$-valued stopping times:

$$\mathcal{A} := \{\tau \in \mathbb{T} : \tau \in \mathcal{T} \cup \{\infty\} \ a.s.\}.$$

We consider American-type put/call options:

$$V_i(s) = \sup_{\tau \in \mathcal{A}} \mathbb{E}_s[e^{-r\tau} G_i(S_\tau) 1_{\{\tau < \infty\}}], \quad i = p, c, \tag{2.1}$$

for

$$G_p(s) := (K - s)^+ \quad \text{and} \quad G_c(s) := (s - K)^+,$$

for a given discount factor $r > 0$ and strike price $K > 0$.

In order to obtain a nontrivial solution, we assume the following for the call option.

Assumption 2.1. For the call option V_c, we assume that $\mathbb{E} S_1 < e^r$.

Remark 2.1 (*Classical Case*). The classical case with the set of admissible strategies $\mathcal{A}$ replaced by $\mathbb{T}$ has been solved by Mordecki (2002) and Alili and Kyprianou (2005). (i) In the classical put case, it is optimal to stop as soon as S goes below the level

$$A^*_{p,\infty} := K \mathbb{E}[\exp(\underline{X}_{\mathbf{e}_r})],$$

where $\underline{X}$ is the running infimum process of X and $\mathbf{e}_r$ is an independent exponential random variable with parameter r. (ii) In the classical call case, under Assumption 2.1, it is optimal to stop as soon as S goes above

$$B^*_{c,\infty} := K \mathbb{E}[\exp(\overline{X}_{\mathbf{e}_r})],$$

where $\overline{X}$ is the running supremum process of X.

3. Optimality of periodic barrier strategies

In this section, we show that the optimal stopping times are of the form

$$\tau_A^- := \inf\{T \in \mathcal{T} : S_T \le A\} \quad \text{and} \quad \tau_B^+ := \inf\{T \in \mathcal{T} : S_T \ge B\} \tag{3.1}$$

for suitably chosen barriers A and B. Here and throughout, we let $\inf \varnothing = \infty$.

Let $\bar{\mathcal{A}} := \{\tau \in \mathbb{T} : \tau \in \bar{\mathcal{T}} \cup \{\infty\} \ a.s.\}$ with $\bar{\mathcal{T}} := \mathcal{T} \cup \{0\}$. Define the value function of an auxiliary problem where immediate stopping is also allowed:

$$\bar{V}_i(s) = \sup_{\tau \in \bar{\mathcal{A}}} \mathbb{E}_s[e^{-r\tau} G_i(S_\tau) 1_{\{\tau < \infty\}}], \quad i = p, c, \quad s > 0. \tag{3.2}$$

By the strong Markov property,

$$V_i(s) = \mathbb{E}_s[e^{-rT_1} \bar{V}_i(S_{T_1})], \quad i = p, c, \quad s > 0.$$