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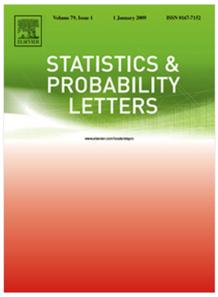

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A NOTE ON THE MONOTONE STOCHASTIC ORDER FOR PROCESSES WITH INDEPENDENT INCREMENTS

DAVID CRIENS

ABSTRACT. We construct a coupling of two processes with independent increments which proves conditions for a monotone stochastic order.

1. Introduction

Stochastic orders for processes are interesting from many perspectives. In mathematical finance, for instance, stochastic orders are used to study monotonicity of option prices, see [3], or to solve problems arising in portfolio optimization, see [7]. Another application lies in the comparison of path-properties of processes. For example, Ikeda and Watanabe use a monotone stochastic order for Itô processes to prove Khasminkii's test for explosion, see [4].

Let us introduce the stochastic order we study in this note. Denote by $\mathbb{D}$ the space of all càdlàg functions $[0, \infty) \to \mathbb{R}$ and equip it with the Skorokod topology. A Borel functional $f\colon \mathbb{D} \to \mathbb{R}$ is called increasing if $f(\omega) \leq f(\alpha)$ for all $\omega, \alpha \in \mathbb{D}$ with $\omega_t \leq \alpha_t$ for all $t \in [0, \infty)$. Let X and Y be two processes with paths in $\mathbb{D}$. We write $X \preceq_{pst} Y$ if

$$E[f(X)] \leq E[f(Y)] \tag{1.1}$$

for all bounded increasing functionals $f\colon \mathbb{D} \to \mathbb{R}$. The boundedness is not crucial, i.e. if the inequality (1.1) holds for all bounded increasing functionals, it holds also for all increasing functionals for which the expectations are well-defined, see [6]. Here, *pst* is an acronym for *pathwise stochastic order*.

A process Z on $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, P)$ is called a process with independent increments (PII) if it is a real-valued càdlàg and adapted process such that $Z_0 = 0$ and for all $0 \leq s \leq t < \infty$ the random variable $Z_t - Z_s$ is independent of $\mathcal{F}_s$. We assume that X and Y are two PIIs.

We introduce some natural conditions for $X \preceq_{pst} Y$: Suppose that X and Y satisfy a drift condition, see (2.3) below, have the same Gaussian components and posses the specific jump structure that the intensity of negative jumps of Y is less than the intensity of negative jumps of X, and the intensity of positive jumps of Y is larger than the intensity of positive jumps of X, see (2.4) below. For Lévy processes and quasi-left continuous PIIs such conditions appear in [1, 2].

Conditions of this type can be proven by a coupling argument, i.e. one constructs copies of X and Y on a common probability space such that a.s. $X_t \leq Y_t$ for all $t \in [0, \infty)$. It is easy to see that such a coupling implies $X \preceq_{pst} Y$.

Let us shortly comment on coupling ideas in the related literature. In [1], a coupling is given for the case where X and Y are compound Poisson processes, whose underlying Poisson processes have the same jump intensities. The idea is to realize X and Y via one common Poisson process. In [2], the PIIs X and Y are decomposed (in law) into two parts, i.e. $X \overset{\text{law}}{=} Z + Z^X$ and $Y \overset{\text{law}}{=} Z + Z^Y$, where $Z^Y - Z^X$ has only positive paths. This decomposition holds (modulo integrability issues) if X and Y have no fixed times of discontinuity.

In this note we present an alternative coupling, which even applies to PIIs with fixed times of discontinuity. The idea is as follows: We start with a PII which has the same intensities of negative jumps as X and the same intensities of positive jumps as Y. In other words, the PII has a higher frequency of jumps than both X and Y. Then, by removing negative jumps independently with a certain rate, we obtain a

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