

Accepted Manuscript

The median of an exponential family and the normal law

G rard Letac, Lutz Mattner, Mauro Piccioni

PII: S0167-7152(17)30316-4
DOI: <https://doi.org/10.1016/j.spl.2017.10.002>
Reference: STAPRO 8043

To appear in: *Statistics and Probability Letters*

Received date : 9 June 2017
Revised date : 3 October 2017
Accepted date : 4 October 2017

Please cite this article as: Letac G., Mattner L., Piccioni M., The median of an exponential family and the normal law. *Statistics and Probability Letters* (2017), <https://doi.org/10.1016/j.spl.2017.10.002>

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.



The median of an exponential family and the normal law

G rard Letac*, Lutz Mattner†, Mauro Piccioni‡

October 18, 2017

Abstract

Let P be a probability on the real line generating a natural exponential family $(P_t)_{t \in \mathbb{R}}$. We show that the property that t is a median of P_t for all t characterizes P as the standard Gaussian law $N(0, 1)$.

KEYWORDS: Characterization of the normal laws, real exponential families, median of a distribution, Choquet-Deny equation.

MSC2010 CLASSIFICATION: 62E10, 60E05, 45E10.

1 Introduction

Let P be a probability on the real line and assume that

$$L(t) = \int_{-\infty}^{+\infty} e^{tx} P(dx) < \infty \quad \text{for } t \in \mathbb{R}. \quad (1)$$

Such a probability generates the natural exponential family

$$\mathcal{F}_P = \{P_t(dx) = \frac{e^{tx}}{L(t)} P(dx), t \in \mathbb{R}\}.$$

Then it might happen that the natural parameter t of $\mathcal{F}_P$ is always a median of P_t , in the sense of

$$P_t((-\infty, t)) \leq \frac{1}{2} \leq P_t((-\infty, t]) \quad \text{for } t \in \mathbb{R}. \quad (2)$$

In the sequel we denote by $\mathcal{P}$ the set of probabilities P such that (1) and (2) are fulfilled. A noteworthy example of an element of $\mathcal{P}$ is the standard normal

*Laboratoire de Statistique et Probabilit s, Universit  Paul Sabatier, Toulouse, France.
gerard.letac@math.univ-toulouse.fr

†Universit t Trier, Fachbereich IV - Mathematik, 54286 Trier, Deutschland.
mattner@uni-trier.de

‡Dipartimento di Matematica, Sapienza Universit  di Roma, 00185 Roma, Italia.
piccioni@mat.uniroma1.it

Download English Version:

<https://daneshyari.com/en/article/7548801>

Download Persian Version:

<https://daneshyari.com/article/7548801>

[Daneshyari.com](https://daneshyari.com)