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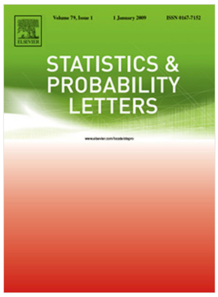

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The median of an exponential family and the normal law

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Abstract

Let P be a probability on the real line generating a natural exponential family $(P_t)_{t\in\mathbb{R}}$. We show that the property that t is a median of P_t for all t characterizes P as the standard Gaussian law $N(0,1)$.

Keywords: Characterization of the normal laws, real exponential families, median of a distribution, Choquet-Deny equation.

MSC2010 classification: 62E10, 60E05, 45E10.

1 Introduction

Let P be a probability on the real line and assume that

$$L(t) = \int_{-\infty}^{+\infty} \mathrm{e}^{tx} P(\mathrm{d}x) < \infty \quad \text{for } t \in \mathbb{R}. \tag{1}$$

Such a probability generates the natural exponential family

$$\mathcal{F}_P = \{P_t(\mathrm{d}x) = \frac{\mathrm{e}^{tx}}{L(t)} P(\mathrm{d}x),\ t \in \mathbb{R}\}.$$

Then it might happen that the natural parameter t of $\mathcal{F}_P$ is always a median of P_t, in the sense of

$$P_t((-\infty, t)) \le \frac{1}{2} \le P_t((-\infty, t]) \quad \text{for } t \in \mathbb{R}. \tag{2}$$

In the sequel we denote by $\mathcal{P}$ the set of probabilities P such that (1) and (2) are fulfilled. A noteworthy example of an element of $\mathcal{P}$ is the standard normal

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