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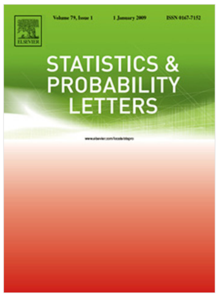

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Some asymptotic results for fiducial and confidence distributions

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Abstract. Under standard regularity assumptions, we provide simple approximations for *specific* classes of fiducial and confidence distributions and discuss their connections with objective Bayesian posteriors. For a real parameter the approximations are accurate at least to order $O(n^{-1})$. For the mean parameter $\boldsymbol{\mu} = (\mu_1, \ldots, \mu_k)$ of an exponential family, our fiducial distribution is asymptotically normal and invariant to the importance ordering of the μ_i's.

Keywords: ancillary statistic, confidence curve, coverage probability, natural exponential family, matching prior, reference prior.

1 Introduction

Confidence and fiducial distributions, often confused in the past, have recently received a renewed attention by statisticians thanks to several contributions which clarify the concepts within a purely frequentist setting and overcome the lack of rigor and completeness typical of the original formulations. For a wide and comprehensive presentation of the theory of confidence distributions and a rich bibliography we refer the reader to the book by Schweder & Hjort (2016) and to the review paper by Xie & Singh (2013). This latter also highlights the importance of this theory in meta-analysis, see also Liu et al. (2015). For what concerns fiducial distributions Hannig and his coauthors, starting from the original idea of Fisher, have developed in several papers a *generalized fiducial inference* which is suitable for a large range of situations; see Hannig et al. (2016) for a complete review on the topic and updated references.

Given a random vector $\mathbf{S}$ (representing the observations or a sufficient statistic) with distribution indexed by $\boldsymbol{\eta} = (\theta, \boldsymbol{\lambda})$, where θ is the real parameter of interest, a *confidence distribution* (CD) for θ is a function C of $\mathbf{S}$ and θ such that: i) $C(\mathbf{s}, \cdot)$ is a distribution function on $\mathbb{R}$ for any fixed realization $\mathbf{s}$ of $\mathbf{S}$ and ii) $C(\mathbf{S}, \theta)$ has a uniform distribution on $(0, 1)$, whatever the true value of $\boldsymbol{\eta}$. The second condition is crucial because it implies that the coverage of the intervals derived from C is exact. If it is satisfied only for the sample size tending to infinity, C is an *asymptotic* CD and the coverage is correct only approximately. Given a CD, it is

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