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Joint arrival process of multiple independent batch Markovian arrival processes

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ABSTRACT

The joint arrival process of multiple independent batch Markovian arrival processes (BMAP) is analyzed. First, the parameter matrices of the joint arrival process are constructed, and their relations with the parameter matrices of the component BMAPs are also obtained. Second, it is proved that the parameter matrices of the joint arrival process have the same properties as the BMAP; and based on these properties, in analyzing the queueing models, some relations for the joint arrival process either can inherit from the existing literature on the BMAP and the multiple-BMAP, or can be derived with the same complexity as the case for the BMAP.

1. Introduction

Queueing models are considered to be effective instruments in the performance evaluation of production and manufacture systems, computer and communication networks, traffic and transportation systems, and social public service systems. By now, various queueing models have been studied extensively.

A classical queueing model consists of four parts, including the arrival process, the server (single server or multiple servers), the queue (single queue or multiple queues), and the service discipline. The queueing models with different characteristics can model different actual systems. And, the researchers are encouraged to construct the general queueing model, which can be used to model many actual systems. To construct a general queueing model, the arrival process with general properties should be used. The batch Markovian arrival process (BMAP) (Lucantoni, 1991) can capture the batch, correlated and bursty nature of the real arrival processes. Moreover, the BMAP includes many familiar arrival processes, such as the Poisson process (M), the PH-renewal process (PH), the Markov-modulated Poisson process (MMPP), the Markovian arrival process (MAP), and so on. In this paper, we will be concerned with the BMAP.

In the literature, the following four cases, in which the BMAP is used as the arrival process, are common. First, in the queueing model with one single queue, the customers arrive to the queue according to the single BMAP, see Dudin et al. (2016) and Ghosh and Banik (2017). Second, in the queueing model with one single queue, several types of customers arrive to the same queue according to different BMAPs, which are governed by the same Markov process with the finite state space, see Masuyama and Takine (2002, 2003). The joint arrival process of these dependent BMAPs is constructed, named the multiple-BMAP, and used to analyze the performance of the corresponding queueing models in Masuyama and Takine (2002, 2003). Third, in the queueing models with one single queue, several types of customers arrive to the same queue

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according to different BMAPs, which are mutually independent. Lucantoni (1993) indicated that the class of BMAP's is closed under superposition. So, for the third case, if some analysis are irrelevant to the types of the customers, the superposition of multiple independent BMAPs can be used to simplify the analysis processes. Otherwise, the superposition cannot be used. Fourth, in the queueing models with multiple queues, several types of customers arrive to the different queues according to different BMAPs, which are mutually independent. This case is common in the polling models, see Saffer and Telek (2010). From Masuyama and Takine (2002, 2003), the appropriate parameter matrices, which characterize the joint arrival process of multiple dependent BMAPs, can bring convenience to the analysis of the corresponding queueing models, because some results and methods can inherit from the existing literature. So far, although multiple independent BMAPs appear as the third and fourth cases, the joint arrival process of these BMAPs has not been considered. Referring to the multiple-BMAP, if the multiple independent BMAPs are considered as a joint arrival process characterized by the appropriate parameter matrices, then this joint arrival process should be able to make it more convenient to analyze the corresponding queueing models. Motivated from this, the joint arrival process of multiple independent BMAPs will be analyzed.

The main contributions of this paper are twofold. First, the parameter matrices of the joint arrival process of K independent BMAPs $\left(K \in \mathrm{N}^+,\ K \geq 2\right)$ are constructed, and their relations with the parameter matrices of the component BMAPs are also obtained. Second, it is proved that the parameter matrices of the joint arrival process have the properties, which are the same as the properties of the BMAP. Based on the properties of the parameter matrices, in analyzing the corresponding queueing models, some relations for the joint arrival process either can inherit from the BMAP and the multiple-BMAP, or can be derived with the same complexity as the case for the BMAP.

Throughout this paper, some notations are used as follows. $\mathrm{N}^m = \underbrace{\mathrm{N} \times \mathrm{N} \times \cdots \times \mathrm{N}}_{m}$, where $m \in \mathrm{N}^+$. $\mathbf{0}$ denotes a vector or matrix of appropriate size consisting of 0's; $\mathbf{0}_{m\times m}$ denotes a $m \times m$ matrix consisting of 0's. $\mathbf{e}$ denotes a column vector of appropriate size consisting of 1's. $\mathbf{I}_m$ denotes a $m \times m$ identity matrix. Given two vectors $\boldsymbol{i}, \boldsymbol{j} \in \mathrm{N}^m$, where $\boldsymbol{i} = (i_1, i_2, \ldots, i_m)$ and $\boldsymbol{j} = (j_1, j_2, \ldots, j_m)$, $\rho(\boldsymbol{i})$ represents the number of non-zero components in the vector $\boldsymbol{i}$; $\rho(\boldsymbol{i}, \boldsymbol{j})$ represents the number of non-zero components in the vector $(\boldsymbol{i} - \boldsymbol{j})$; $\boldsymbol{i} = \boldsymbol{j}$ if and only if $\rho(\boldsymbol{i}, \boldsymbol{j}) = 0$; $\boldsymbol{i} \neq \boldsymbol{j}$ if and only if $\rho(\boldsymbol{i}, \boldsymbol{j}) > 0$; $\boldsymbol{i} \leq \boldsymbol{j}$ represents $i_k \leq j_k$ for any $k \in \mathbb{M}$, where $\mathbb{M} = \{1, 2, \ldots, m\}$; $\boldsymbol{i} < \boldsymbol{j}$ represents that there are at least one $k \in \mathbb{M}$ such that $i_k < j_k$ under the condition $\boldsymbol{i} \leq \boldsymbol{j}$; $\boldsymbol{i} \geq 0$ represents $i_k \geq 0$ for any $k \in \mathbb{M}$; $\boldsymbol{i} > 0$ represents $i_k > 0$ for any $k \in \mathbb{M}$. Given two sets A and B, $A \setminus B = \{x | x \in A \text{ and } x \notin B\}$. Given a matrix $\mathbf{M}$ whose elements are indexed by $(i, j) \in \Omega_1 \times \Omega_2$, where the set Ω_1 consists of the row indices which are all either scalars or row vectors with the same dimension, and the set Ω_2 consists of the column indices which are all either scalars or row vectors with the same dimension, $\mathbf{M}$ can be denoted by $\mathbf{M} = \left(\mathbf{M}_{i,j} : i \in \Omega_1, j \in \Omega_2\right)$, where $\mathbf{M}_{i,j}$ represents the (i, j)th element, and no matter the indices are scalars or vectors, the elements in each row (column) are arranged in the lexicographical order among the corresponding row (column) indices.

The rest of this paper is organized as follows. In Section 2, the preliminary about the BMAP is given. In Section 3, the joint arrival process of multiple independent BMAPs is characterized, and then the application of the characterization of the joint arrival process is demonstrated.

2. Preliminary

The BMAP was proposed by Lucantoni (1991), and it was described by an irreducible Markov process. In Lucantoni et al. (1990), Lucantoni et al. described the MAP by a Markov process with an absorbing state. Referring to Lucantoni et al. (1990), in this section, the BMAP will be redescribed based on the Markov process with an absorbing state, since compared with the irreducible Markov process, the BMAP can be better understood, if it is described by the Markov process with an absorbing state. Notice that no matter which Markov process is applied, the given parameter matrices of the BMAP are coincident.

In this paper, K BMAPs are considered, where $K \in \mathrm{N}^+$ and $K \geq 2$. And assume that these BMAPs are mutually independent. The kth BMAP is denoted by the BMAP-k, where $k \in \mathbb{K}$ and $\mathbb{K} = \{1, 2, \ldots, K\}$. Without loss of generality, the BMAP-k is described in the following, and we use "(k)" as the superscript for the notations corresponding to the BMAP-k.

Given a Markov process $X^{(k)}$ on the state space $S^{(k)}$, where $S^{(k)} = M^{(k)} \cup \left\{m^{(k)} + 1\right\}$, $m^{(k)} \in \mathrm{N}^+$, $M^{(k)} = \left\{1, 2, \ldots, m^{(k)}\right\}$, the states in $M^{(k)}$ are transient and the state $m^{(k)} + 1$ is absorbing, suppose that $X^{(k)}$ has the following characteristics.

(1) The sojourn time in any state $i \in M^{(k)}$ is exponentially distributed with the parameter $\lambda_i^{(k)}$ $(0 < \lambda_i^{(k)} < \infty)$.
(2) From any state in $M^{(k)}$, the absorption eventually occurs with probability one. After the absorption, the Markov process $X^{(k)}$ immediately restarts in a transient state with some probability, which depends on the state from which the absorption occurred.
(3) The states in $M^{(k)}$ can be reached from each other, by going through the state $m^{(k)} + 1$ or not.

A batch arrival occurs at each time when the Markov process $X^{(k)}$ restarts from the absorbing state. Let $p_{i,j}^{(k)}(n)$, $n \in \mathrm{N}^+$, denote the conditional probability that given the current state i, the Markov process $X^{(k)}$ will move to the absorbing state $m^{(k)} + 1$ at the next transition epoch and then immediately restart from the state j, accompanied by a batch arrival of size n, where $0 \leq p_{i,j}^{(k)}(n) \leq 1$, $i, j \in M^{(k)}$. Let $p_{i,j}^{(k)}(0)$ denote the conditional probability that given the current state i, the Markov process $X^{(k)}$ will move to the state j at the next transition epoch, accompanied by no arrivals, where $0 \leq p_{i,j}^{(k)}(0) \leq 1$, $i, j \in M^{(k)}$, $i \neq j$. It is meaningless to discuss the event that given the current state i $(i \in M^{(k)})$, the Markov process $X^{(k)}$ will move to the state i at the next transition epoch, accompanied by no arrivals, but for convenience of analysis, let $p_{i,i}^{(k)}(0)$ denote