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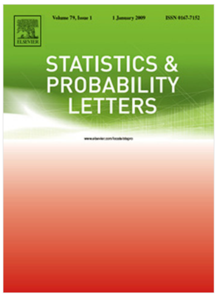

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A Large Deviation Inequality for β-mixing Time Series and its Applications to the Functional Kernel Regression Model

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Abstract

We give a new large deviation inequality for sums of random variables of the form $Z_k = f(X_k, X_t)$ for $k, t \in \mathbb{N}$, t fixed, where the underlying process X is β-mixing. The inequality can be used to derive concentration inequalities. We demonstrate its usefulness in the functional kernel regression model of Ferraty et al. (2007) where we study the consistency of dynamic forecasts.

Keywords: Asymptotic inequalities, β-mixing, Functional data analysis, Large deviation inequality, Nonparametric statistics, Time series
2010 MSC: Primary: 62G20, 62M10, 37A25, Secondary: 62G09

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1. Introduction

In this article we study asymptotic bounds for the probability

$$\mathbb{P}\left(n^{-1} \left| \sum_{k=1}^{n} f(X_k, X_t) - \mathbb{E}\left[\, f(X_0, x)\,\right]\big|_{x=X_t} \right| \geq \varepsilon \right) \text{ where } t \in \mathbb{Z} \tag{1}$$

for a real-valued function f and for a stationary stochastic process $\{X_k : k \in \mathbb{Z}\}$ which takes values in a state space $(S, \mathfrak{S})$. If the function f is bounded by $B > 1$, we obtain for a certain constant c an exponential rate of convergence for (1) which is in $O(\exp(-c\varepsilon n/(B \log n \log\log n)))$. So modulo a logarithmic factor which comes from the dependence in the data, the rate corresponds to the rates of classical large deviations inequalities for independent random variables.

Large deviation inequalities are a major tool for the asymptotic analysis in probability theory and statistics. One of the first inequalities of this type was published by Bernstein (1927) who considers the case $\mathbb{P}(|S_n| > \varepsilon)$, where $S_n = \sum_{k=1}^{n} X_k$ for bounded real-valued random variables $X_1, \ldots, X_n$ which are i.i.d. and have expectation zero. There are various versions and generalizations of Bernstein's inequality, e.g., Hoeffding (1963). In particular, deviation inequalities for dependent data such as stochastic processes are nowadays important: Bernstein inequalities for time series are developed in Carbon (1983), Collomb (1984), Bryc and Dembo (1996) and Merlevède et al. (2009). Arcones (1995) develops Bernstein-type inequalities for U-statistics. Valenzuela-Domínguez et al. (2017) give a further generalization to strong mixing random fields $\{X_s : s \in \mathbb{Z}^N\}$ which are defined on the regular lattice $\mathbb{Z}^N$ for some lattice dimension $N \in \mathbb{N}_+$. A similar version for independent multivariate random variables is given by Ahmad and Amezziane (2013). Krebs (2017) gives a Bernstein inequality for strong mixing random fields which are defined on

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