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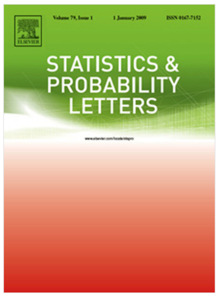

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CHARACTERIZING THE PATH-INDEPENDENT PROPERTY OF THE GIRSANOV DENSITY FOR DEGENERATED STOCHASTIC DIFFERENTIAL EQUATIONS

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Abstract. In this paper, we derive a characterisation theorem for the path-independent property of the density of the Girsanov transformation for *degenerated* stochastic differential equations (SDEs), extending the characterisation theorem of [13] for the non-degenerated SDEs. We further extends our consideration to non-Lipschitz SDEs with jumps and with degenerated diffusion coefficients, which generalises the corresponding characterisation theorem established in [10].

1. Introduction

Let $(\Omega, \mathcal{F}, \mathbb{P}, \{\mathcal{F}_t\}_{t\geqslant 0})$ be a filtered probability space. Let $d, m \in \mathbb{N}$ be fixed. We are concerned with the following SDE

$$\mathrm{d}X_t = b(t, X_t)\mathrm{d}t + \sigma(t, X_t)\mathrm{d}W_t, \quad t \geq 0 \tag{1}$$

where

$$b : [0, \infty) \times \mathbb{R}^d \to \mathbb{R}^d, \quad \sigma : [0, \infty) \times \mathbb{R}^d \to \mathbb{R}^{d\otimes m}$$

$(W_t)_{t\geqslant 0}$ is an m-dimensional $\{\mathcal{F}_t\}_{t\geqslant 0}$-Brownian motion. Under standard usual conditions, e.g. the two coefficients b and σ satisfy linear growth and local Lipschitz conditions (for the second variable), there is a unique solution to the above SDE (1) for a given initial data X_0, see, e.g., [3].

The celebrated Girsanov theorem provides a very powerful tool to solve SDEs under the name of the *Girsanov transformation* or the *transformation of the drift*. We use $|\cdot|$ and $\langle \cdot, \cdot \rangle$ to denote the Euclidean norm and scalar product of vectors in $\mathbb{R}^m$ or $\mathbb{R}^d$, respectively. Let $\gamma : [0, \infty) \times \mathbb{R}^d \to \mathbb{R}^m$ be a measurable function such that the following exponential integrability along the paths of the solution $(X_t)_{t\geqslant 0}$ holds (also known as Novikov condition)

$$\mathbb{E}\left(\exp\left\{-\int_0^t |\gamma(s, X_s)|^2 \mathrm{d}s + \int_0^t \langle \gamma(s, X_s), \mathrm{d}W_s\rangle\right\}\right) < \infty, \quad t \geqslant 0. \tag{2}$$

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Keywords: degenerated stochastic differential equations (SDEs), Girsanov transformation, non-Lipschitz SDEs with jumps, semi-linear partial integro-differential equation of parabolic type.

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