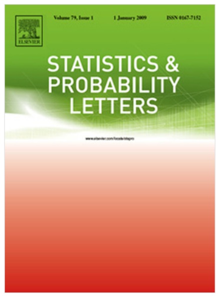

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An Extension of Feller's Strong Law of Large Numbers

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Abstract This paper presents a general result that allows for establishing a link between the Kolmogorov-Marcinkiewicz-Zygmund strong law of large numbers and Feller's strong law of large numbers in a Banach space setting. Let $\{X, X_n; n \geq 1\}$ be a sequence of independent and identically distributed Banach space valued random variables and set $S_n = \sum_{i=1}^{n} X_i,\ n \geq 1$. Let $\{a_n; n \geq 1\}$ and $\{b_n; n \geq 1\}$ be increasing sequences of positive real numbers such that $\lim_{n\to\infty} a_n = \infty$ and $\{b_n/a_n;\ n \geq 1\}$ is a nondecreasing sequence. We show that

$$\frac{S_n - n\mathbb{E}\left(XI\{\|X\| \leq b_n\}\right)}{b_n} \to 0 \ \text{ almost surely}$$

for every Banach space valued random variable X with $\sum_{n=1}^{\infty} \mathbb{P}(\|X\| > b_n) < \infty$ if $S_n/a_n \to 0$ almost surely for every symmetric Banach space valued random variable X with $\sum_{n=1}^{\infty} \mathbb{P}(\|X\| > a_n) < \infty$. To establish this result, we invoke two tools (obtained recently by Li, Liang, and Rosalsky): a symmetrization procedure for the strong law of large numbers and a probability inequality for sums of independent Banach space valued random variables.

Keywords Feller's strong law of large numbers · Kolmogorov-Marcinkiewicz-Zygmund strong law of large numbers · Rademacher type p Banach space · Sums of independent random variables

Mathematics Subject Classification (2000): 60F15 · 60B12 · 60G50

Running Head: On Feller's Strong Law of Large Numbers

1 Introduction and the main result

We begin with stating Feller's (1946) strong law of large numbers (SLLN) as follows.

Theorem A. (Feller's SLLN. Theorems 1 and 2 of Feller (1946)). *Let $\{X, X_n; n \geq 1\}$ be a sequence of independent and identically distributed (i.i.d.) real-valued random variables, and let $S_n = \sum_{i=1}^{n} X_i$, $n \geq 1$. Let $\{b_n; n \geq 1\}$ be an increasing sequence of positive real numbers. Suppose that one of the following two sets of conditions holds:*
(i) *For some $0 < \delta < 1$, $\mathbb{E}|X|^{1+\delta} = \infty$, $\mathbb{E}X = 0$, and there exists an ϵ with $0 \leq \epsilon < 1$ such that*

$$b_n n^{-1/(1+\epsilon)} \uparrow \quad \text{and} \quad b_n/n \downarrow,$$

(ii) $\mathbb{E}|X| = \infty$ *and*

$$b_n/n \uparrow .$$

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