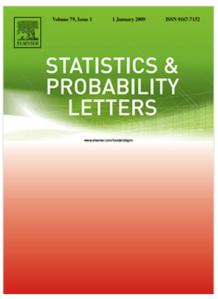
Accepted Manuscript



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PII: S0167-7152(17)30294-8
DOI: https://doi.org/10.1016/j.spl.2017.09.008
Reference: STAPRO 8029

To appear in: *Statistics and Probability Letters*

Received date : 26 March 2017
Revised date : 13 September 2017
Accepted date : 14 September 2017

Please cite this article as: Sun X., Guo F., On moment estimates and continuity for solutions of SDEs driven by fractional Brownian motions under non-Lipschitz conditions. *Statistics and Probability Letters* (2017), https://doi.org/10.1016/j.spl.2017.09.008

On moment estimates and continuity for solutions of SDEs driven by fractional Brownian motions under non-Lipschitz conditions

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Abstract

In this paper, we consider stochastic differential equations (SDEs) driven by fractional Brownian motions under non-Lipschitz conditions. We give the p-th moment estimates for solutions of considered SDEs by divergence-type Itô formula. Furthermore, we study the continuity for solutions of the considered SDEs.

Keywords: p-th moment estimates, divergence, Itô formula, fractional Brownian motion

1. Introduction

Stochastic differential equations (SDEs) driven by Brownian motions are widely used in practice, such as physical systems, finance and economic areas. A lot of interest has been attracted to the study of solutions of such SDEs. For instance, the existence, uniqueness and p-th moment estimates of solutions of such SDEs are investigated under Lipschitz conditions in Mao (2007) and under non-Lipschitz conditions in Ren and Xia (2009); Wei and Wang (2007); Kim (2014).

Recently, fractional Brownian motion with Hurst parameter $H > 1/2$ is widely used in practice. Fractional Brownian motion was first introduced in Kolmogorov (1940) and later proved in Mandelbrot and Van Ness (1968) to have a stochastic integration representation of Brownian motion. Since fractional Brownian motion is not a semi-martingale, the stochastic integral for such motion is different from the Itô integral. For more details of integrals for fractional Brownian motion, see Biagini et al. (2008); Decreusefond and Üstünel (1999); Nualart and Răşcanu (2002); Duncan et al. (2000). There are considerable amount of interesting work on SDEs driven by fractional Brownian motions which are extensively used in biological engineering, chemistry and chemical engineering. Under Lipschitz conditions, the existence, uniqueness and continuity of the local solutions are established in Xu et al. (2008) by means of Driver theorem and Picard iteration. The case under non-Lipschitz conditions is considered in Pei and Xu (2016). In this paper, we focus on the p-th moment estimates (Theorem 3.4) and continuity (Theorem 3.5) of solutions for such SDEs under non-Lipschitz conditions.

2. Preliminaries

A Gaussian process $(B^H(t))_{t\geq 0}$ is called a fractional Brownian motion with Hurst parameter $H > 1/2$ if it has mean zero and the covariance function

$$R(s,t) = \mathbb{E}[B^H(t)B^H(s)] = \frac{1}{2}(t^{2H} + s^{2H} - |t-s|^{2H}) = \alpha_H \int_0^t \int_0^s |r-u|^{2H-2} du dr,$$

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