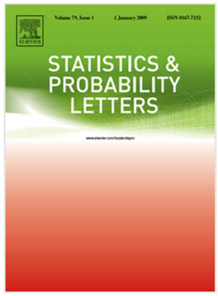

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Numerical Computation of Hitting Time Distributions of Increasing Lévy Processes

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Abstract

We introduce a method for computation of hitting time distribution of an increasing Lévy process using the inverse Fourier transform and the Hilbert transform under the assumption that the characteristic function of the process is given. Its efficiency is demonstrated by the Kolmogorov–Smirnov test.

Keywords: Lévy Process, Hitting time, Characteristic function, Hilbert transform, Sinc expansion, Lambert function
2010 MSC: 60E10, 60G51, 65R99

1. Introduction

A process $V(t)$, $t \geq 0$, is called a Lévy process if (i) $V(0) = 0$ almost surely, (ii) V has independent and stationary increments, (iii) V is stochastically continuous, i.e., for $a > 0$ and $s > 0$, $\lim_{t\to s} \mathbb{P}\{|V(t) - V(s)| > a\} = 0$. A process is said to be increasing if sample paths are increasing almost surely and adapted. An increasing Lévy process is called a subordinator. (See Applebaum (2009) or Bertoin (1999).) For a given L the hitting time of a process $V(t)$ is defined by $T(L) = \inf\{t > 0 : V(t) \geq L\}$. We introduce a new numerical method of finding a hitting time distribution of an increasing Lévy process under the assumption that the characteristic function of the process is given.

Hitting times of a standard Brownian motion and a Brownian motion with drift are subordinators, respectively called a Lévy subordinator and an inverse Gaussian subordinator. Decreusefond and Nualart (2008) derived a hitting time of a general Gaussian process. Using the Laplace transform, Meerschaert and Scheffler (2006), Meerschaert and Scheffler (2008), Veillette and Taqqu (2010a) and Vellaisamy and Kumar (2015) derived the density formula of hitting time process of a subordinator under various conditions; and Veillette and Taqqu (2010b) gave a numerical method. However, the approximation error was not estimated.

In this paper, we use the Fourier transform to compute the distribution of a hitting time of a subordinator, and apply the Fourier inversion formula to obtain the distribution function. In doing so, we employ the sinc expansion combined with the Hilbert transform. The approximation formula of the distribution function and the error are obtained. To simplify the approximation error, we choose sufficient truncation level and discretization level which vary with time. Its efficiency is tested using the Kolmogorov–Smirnov test for some practical examples.

In Section 2 the numerical method is discussed for using the Hilbert transform combined with the sinc expansion. In Section 3, we compute the hitting time distribution of the one-sided tempered stable subordinator such as an inverse Gaussian process. Section 4 concludes the paper.

2. Main Results

Let $V(t)$ be an increasing process with density function $p(L,t)$, i.e., $\mathbb{P}\{V(t) \leq L\} = \int_{-\infty}^{L} p(\ell,t)\mathrm{d}\ell$ for each t. For a given L let $T(L) = \inf\{t > 0 : V(t) \geq L\}$ be the hitting time process with density function $q(L,t)$, i.e., $\mathbb{P}\{T(L) \leq t\} =$

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