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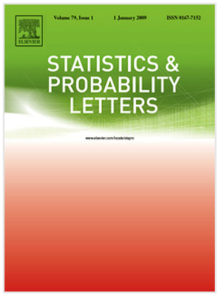

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Convergence rate in precise asymptotics for Davis law of large numbers

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Abstract

Let $\{X, X_n, n \geq 1\}$ be a sequence of i.i.d. random variables with $\mathbb{E}[X] = 0$ and $\mathbb{E}[X^2] = \sigma^2 \in (0, \infty)$, and set $S_n = \sum_{k=1}^n X_k, n \geq 1$. For any $\delta \geq 0$, let

$$\gamma_\delta = \lim_{n\to\infty} \Big(\sum_{j=1}^n \frac{(\log j)^\delta}{j} - \frac{(\log n)^{\delta+1}}{\delta+1} \Big) \text{ and } \eta_\delta = \sum_{n=1}^\infty \frac{(\log n)^\delta}{n} \mathbb{P}(S_n = 0).$$

Under the moment condition $\mathbb{E}\Big[X^2\big(\log(1+|X|)\big)^{1+\delta}\Big] < \infty$, we prove that

$$\lim_{\epsilon\searrow 0} \Big[\sum_{n=1}^\infty \frac{(\log n)^\delta}{n} \mathbb{P}\Big(|S_n| \geq \epsilon\sqrt{n\log n}\Big) - \frac{\mathbb{E}\big[|N|^{2\delta+2}\big]}{\delta+1} \sigma^{2\delta+2} \epsilon^{-(2\delta+2)} \Big] = \gamma_\delta - \eta_\delta,$$

which refines Theorem 3 of Gut and Spătaru (2000a).

Keywords: Convergence rate; Precise asymptotics; Davis law of large numbers

1. Introduction

Let $\{X, X_n, n \geq 1\}$ be a sequence of independent and identically distributed (i.i.d.) random variables, and set $S_n = \sum_{k=1}^n X_k$, $n \geq 1$. Baum and Katz (1965) showed that, for $0 < p < 2$ and $r \geq p$,

$$\sum_{n=1}^\infty n^{\frac{r}{p}-2} \mathbb{P}(|S_n| \geq \epsilon n^{\frac{1}{p}}) < \infty \quad \text{for any } \epsilon > 0, \tag{1.1}$$

if and only if $\mathbb{E}[|X|^r] < \infty$, and, when $r \geq 1$, $\mathbb{E}[X] = 0$. For $r = 2$ and $p = 1$, (1.1) reduces to the famous Hsu-Robbins-Erdős Theorem, see Erdős (1949, 1950) and Hsu and Robbins (1947). It follows from the central limit theorem that the sum in (1.1) does not converge when $r = p = 2$. Hence, Davis (1968) replaced $n^{\frac{1}{p}}$ in (1.1) by $\sqrt{n\log n}$ and got that for any $\epsilon > 0$

$$\sum_{n=1}^\infty \frac{\log n}{n} \mathbb{P}\Big(|S_n| \geq \epsilon\sqrt{n\log n}\Big) < \infty, \tag{1.2}$$

if and only if

$$\mathbb{E}[X] = 0 \text{ and } \mathbb{E}[X^2] < \infty. \tag{1.3}$$

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