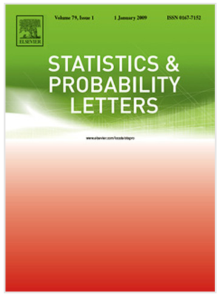
Accepted Manuscript



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PII: S0167-7152(16)30161-4
DOI: http://dx.doi.org/10.1016/j.spl.2016.08.016
Reference: STAPRO 7694

To appear in: *Statistics and Probability Letters*

Received date: 21 April 2016
Revised date: 25 August 2016
Accepted date: 26 August 2016

Please cite this article as: Pelekis, C., Ramon, J., A lower bound on the probability that a binomial random variable is exceeding its mean. *Statistics and Probability Letters* (2016), http://dx.doi.org/10.1016/j.spl.2016.08.016

A lower bound on the probability that a binomial random variable is exceeding its mean

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Abstract

We provide a lower bound on the probability that a binomial random variable is exceeding its mean. Our proof employs estimates on the mean absolute deviation and the tail conditional expectation of binomial random variables.

Keywords: lower bounds, binomial tail, mean absolute deviation, tail conditional expectation, hazard rate order

1. Prologue, related work and main result

Given a positive integer n and a real number $p \in (0,1)$, we denote by $\text{Bin}(n,p)$ a binomial random variable of parameters n and p. Here and later, given two random variables X, Y, the notation $X \sim Y$ will indicate that X and Y have the same distribution. The main purpose of this note is to illustrate that estimates on the mean absolute deviation of a binomial random variable yield a lower bound on $\mathbb{P}[X \geq np]$, where $X \sim \text{Bin}(n,p)$. It should come as no surprise that there exists general machinery that can be employed to such a problem. For example, using Cauchy-Schwartz inequality one can show that, for any random variable Z whose mean equals zero, it holds

$$\mathbb{P}[Z \geq 0] \geq \frac{1}{4} \cdot \frac{\{\mathbb{E}[|Z|]\}^2}{\mathbb{E}[Z^2]} \qquad (1)$$

and the bound can be improved further under information on higher moments (see Veraar [11]). If we now let $Z = X - np$, where $X \sim \text{Bin}(n,p)$, then (1)

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