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A lower bound on the probability that a binomial random variable is exceeding its mean

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Abstract

We provide a lower bound on the probability that a binomial random variable is exceeding its mean. Our proof employs estimates on the mean absolute deviation and the tail conditional expectation of binomial random variables.

Keywords: lower bounds, binomial tail, mean absolute deviation, tail conditional expectation, hazard rate order

1. Prologue, related work and main result

Given a positive integer n and a real number $p \in (0, 1)$, we denote by $\text{Bin}(n, p)$ a binomial random variable of parameters n and p . Here and later, given two random variables X, Y , the notation $X \sim Y$ will indicate that X and Y have the same distribution. The main purpose of this note is to illustrate that estimates on the mean absolute deviation of a binomial random variable yield a lower bound on $\mathbb{P}[X \geq np]$, where $X \sim \text{Bin}(n, p)$. It should come as no surprise that there exists general machinery that can be employed to such a problem. For example, using Cauchy-Schwartz inequality one can show that, for any random variable Z whose mean equals zero, it holds

$$\mathbb{P}[Z \geq 0] \geq \frac{1}{4} \cdot \frac{\{\mathbb{E}[|Z|]\}^2}{\mathbb{E}[Z^2]} \quad (1)$$

and the bound can be improved further under information on higher moments (see Veraar [11]). If we now let $Z = X - np$, where $X \sim \text{Bin}(n, p)$, then (1)

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