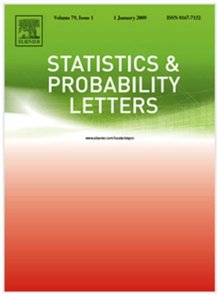

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Tail probability estimates for additive functionals

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Abstract

In this paper, based on techniques of Malliavin calculus, we obtain an explicit bound for tail probabilities of a general class of additive functionals. Applications to fractional Brownian motion and Cox-Ingersoll-Ross process are given to illustrate the theory.

Keywords: Tail probabilities, Additive functionals, Malliavin caculus.
2010 MSC: 60G22, 60H07, 91G30.

1. Introduction

A fractional Brownian motion (fBm) of Hurst parameter $H \in (0,1)$ is a centered Gaussian process $B^H = (B^H_t)_{t\geq 0}$ with covariance function

$$R_H(t,s) := \mathrm{E}[B^H_t B^H_s] = \frac{1}{2}(t^{2H} + s^{2H} - |t-s|^{2H}).$$

It is known that B^H_t admits the so-called Volterra representation (see e.g. [11] pp. 277-279)

$$B^H_t = \int_0^t K_H(t,s)dW_s, \tag{1.1}$$

where $(W_t)_{t\geq 0}$ is a standard Brownian motion, the Volterra kernel $K_H(t,s)$ is defined by

$$K_H(t,s) = C_H\left[\frac{t^{H-\frac{1}{2}}}{s^{H-\frac{1}{2}}}(t-s)^{H-\frac{1}{2}} - (H-\frac{1}{2})\int\limits_s^t \frac{u^{H-\frac{3}{2}}}{s^{H-\frac{1}{2}}}(u-s)^{H-\frac{1}{2}}du\right],\ s\leq t,$$

where C_H is a constant depending only on H.

Motivation of this paper comes from a result given by Nourdin and Viens in [10], where they used their new theory to obtain an upper bound for tail probabilities of fractional Brownian additive functionals

$$\int_0^1 Q(B^H_s)ds, \tag{1.2}$$

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