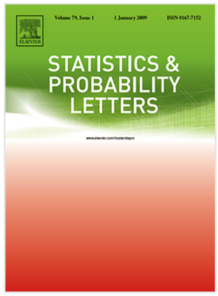
Accepted Manuscript



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PII: S0167-7152(16)30173-0
DOI: http://dx.doi.org/10.1016/j.spl.2016.09.010
Reference: STAPRO 7708

To appear in: *Statistics and Probability Letters*

Received date: 20 January 2016
Revised date: 11 September 2016
Accepted date: 11 September 2016

Please cite this article as: Ruiz, P.A., Rakitko, A., The limit theorem for maximum of partial sums of exchangeable random variables. *Statistics and Probability Letters* (2016), http://dx.doi.org/10.1016/j.spl.2016.09.010

The limit theorem for maximum of partial sums of exchangeable random variables

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Abstract

We obtain the analogue of the classical result by Erdös and Kac on the limiting distribution of the maximum of partial sums for exchangeable random variables with zero mean and variance one. We show that, if the conditions of the central limit theorem of Blum et al. hold, the limit coincides with the classical one. Under more general assumptions, the probability of the random variables having conditional negative drift appears in the limit.

Keywords: exchangeable random variables, limit theorem, maximum, de Finetti's theorem
MSC: 60G09, 60F05, 60G70

1. Introduction

Erdös and Kac established in [14] some fundamental results on the distribution of the maximum of partial sums $S_k := \sum_{i=1}^{k} X_i$, where $\{X_n\}_{n\in\mathbb{N}}$ is a sequence of independent, identically distributed (i.i.d.) centered random variables with variance one. In particular, they proved that the limiting distribution of $n^{-\frac{1}{2}} \max_{1\leq k\leq n} S_k$ is given by $(2\Phi(x)-1)1_{[0,\infty)}(x)$, where $\Phi(\cdot)$ denotes the probability distribution function (p.d.f.) of the standard normal distribution.

Our interest in studying the (rescaled) maximum of partial sums is motivated by its manifold applications. On the one hand, it is directly related to first passage times of random walks and renewal theory [17, 23]. On the other hand, in the classical i.i.d. setting, this statistic has since long been employed in numerous research areas such as hydrology [7], reservoir storage [18] and change-point analysis [19]. Moreover, as a matter of study in extreme value theory, this type of limit theorems are of especial relevance, for instance in finance (see [21] and references therein).

The purpose of this paper is to generalize the original result of Erdös and Kac to exchangeable sequences of random variables and thereby extend the mentioned statistic to further stochastic models. Exchangeable random variables, introduced by de Finetti in [12], are random variables with the property of being conditionally independent. Equivalently, one can think of them as mixtures of i.i.d. random variables directed by a random measure. The study of classical results of probability theory in the exchangeable setting started with the Central Limit Theorem (CLT)