Applied Acoustics 73 (2012) 37–42

Contents lists available at ScienceDirect

Applied Acoustics

journal homepage: www.elsevier.com/locate/apacoust

Predicting insertion loss of large duct systems above the plane wave cutoff frequency

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ARTICLE INFO

Article history:
Received 24 March 2010
Received in revised form 8 February 2011
Accepted 5 July 2011
Available online 29 July 2011

Keywords:
Silencers
Attenuation
Finite element method
Insertion loss

ABSTRACT

If the dimensions of a silencer or muffler component are small compared to an acoustic wavelength, plane wave propagation can be assumed. This is not the case for HVAC (heating, ventilation, and air conditioning) duct systems, and large diesel engine mufflers commonly used in ship and generator sets. For such applications, the wave behavior in the inlet and outlet ducts is three-dimensional. In this paper, the finite element method is utilized to simulate large duct systems with an aim to predict the insertion loss. The boundary condition on the source side is a diffuse field applied by determining a suitable cross-spectral force matrix of the excitation. At the termination, the radiation impedance is calculated utilizing a wavelet algorithm. Simulation results are compared to published measurement results for HVAC plenums and demonstrate good agreement.

1. Introduction

Acoustic waves propagate in ducts in a wide range of applications. For small mufflers and silencers, the duct cross-sectional dimensions are normally small compared to the acoustic wavelength simplifying the analysis since plane wave models [1] are appropriate up to some cutoff frequency. Transfer matrix theory [1] is appropriate in such cases. The plane wave cutoff frequency can be estimated easily for square duct cross-sections and is equal to $c/2d$ where c is the speed of sound and d is a characteristic dimension of the duct cross-section. Similarly, Eriksson [2] showed that the cutoff frequency for a circular duct is $c/1.71d$.

There have been numerous investigations where transfer matrix theory is extended beyond the cutoff frequency to include three-dimensional effects. Most of these investigations were predicated on plane wave behavior in the inlet and outlet ducts. One of the first models of this type was developed by Cummings [3] for rectangular plena. Similarly, Ih and Lee [4,5] and Yi and Lee [6,7] modeled a variety of circular expansion chamber geometries. Ih [8] also investigated rectangular unlined plenum chambers. Munjal [9], Selamet and Radavich [10], and Selamet and Ji [11] used similar approaches to investigate expansion chambers of different geometries. Most recently, Venkateshamb et al. [12] used Green's functions expressed in terms of the rectangular cavity modes to model rectangular expansion chambers. The aforementioned papers [3–12] document useful models for extending plane wave based transfer matrix theory to include muffler components (expansion chambers) which exhibit three dimensional wave behaviors. However, plane wave behavior was assumed in the inlet and outlet ducts to the muffler components.

In the same way, more advanced numerical methods like the finite and boundary element methods have been used. Craggs developed axisymmetric finite element models for reactive [13] and dissipative [14] mufflers. Subsequently, Peat [15] and Sahasrabudhe et al. [16] used three-dimensional finite element models to determine the transfer matrices for muffler components. More recently, Barbieri et al. [17] used the more efficient improved four-pole method [18–21] to determine the four-pole parameters and transmission loss. In each of these investigations, the analyses were performed at frequencies low enough that plane wave behavior was assumed in the inlet and outlet ducts.

The boundary element method has also been used to evaluate transmission loss. Wang et al. [22] used a three-dimensional boundary element method to determine the four-pole parameters and transmission loss, and Wu et al. [21] adopted the improved four-pole method [18–21] for the same purpose. More recently, Herrin et al. [23] used the improved four-pole method to simulate plena at low frequencies and compared the results to published measurements with good agreement.

In all of the aforementioned finite and boundary element studies [13–23], results were reported at frequencies such that plane wave behavior was present in both inlet and outlet ducts. In most of the studies, the four-pole parameters were determined as a precursor to determining transmission loss.

The strategy documented in the current paper differs from the prior studies in several respects. The foremost difference is that

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0003-682X/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.apacoust.2011.07.001

the method utilized allows for three-dimensional wave behavior in the inlet and/or outlet ducts. Similar to the investigations in Refs. [13–15], the finite element method is utilized to simulate the silencer, and the inlet and outlet ducts. The primary difference in this investigation is that the boundary condition on the source side is a diffuse sound field and the termination impedance is determined via a wavelet algorithm.

Another important difference is that insertion loss is used as a performance metric instead of the transmission loss. The transmission loss (TL) is the ratio of the incident (W_i) to the transmitted power (W_t) and can be expressed as

$$TL = 10\log_{10}\frac{W_i}{W_t} \tag{1}$$

Transmission loss of mufflers and silencers is restricted to the attenuation of the component itself and does not include the effects of source and termination impedance.

On the other hand, the insertion loss of a sound attenuating element is the decrease in sound power or sound pressure when the element is inserted into the sound transmission path between source and receiver. It depends upon source and termination characteristics, as well as the attenuation of individual elements. Insertion loss of a multi-component system will only be equal to the transmission loss of the system if both the source and termination impedances are anechoic.

Above the cutoff frequency, transmission loss is problematic as a metric. Measurement and especially prediction of transmission loss is complicated by cross modes in the inlet and/or outlet ducts and depends on the nature of the source (plane wave, diffuse field, etc.). Insertion loss is not necessarily easier to predict using mathematical or computational approaches because source and termination impedances may in fact vary across the duct cross-section. However, insertion loss is preferred for large duct systems because it is easier to measure.

Accordingly, a predictive model above the cutoff frequency should be one in which the source and termination characteristics are random or statistical in nature. Certainly, Monte Carlo methods might be used. However, techniques developed by Langley and Shorter [24–28] and implemented in the ESI VA-One software [29] seem to be better-suited to this particular problem. Using the suggested approach, the modal character of the duct system is described using a finite element (FE) model whereas the boundary conditions at the source and termination are modeled using energy principles. This paper examines the suitability of the approach for large HVAC duct systems.

It should be noted that there are two classic models for analyzing HVAC plena above the cutoff frequency. The first is Wells [30] model based on room acoustics theory, and the second is by Cummings [3] who extended the Wells model to include directivity between the inlet and outlet ducts. Neither approach accounted for the modes in the inlet and outlet ducts, or the plenum itself whereas the method presented in this paper includes the modal behavior of the combined duct and plenum system.

2. Method

The method presented in the discussion which follows is not fundamentally new. Some aspects of the modeling approach have been published elsewhere, but have been applied in different contexts. The discussion details the methodology and references the relevant papers.

The modeling technique utilized is illustrated in Fig. 1. An FEM subsystem is used to simulate the plenum, and inlet and outlet ducts. The input is assumed to be a diffuse acoustic field. A baffled termination is assumed. These boundary conditions are now considered in more detail.

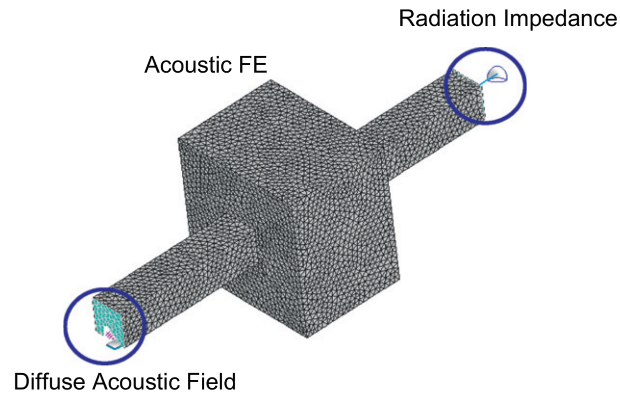

Fig. 1. Schematic illustrating modeling approach.

The diffuse acoustic field loading is applied via a reciprocity relationship between direct field radiation and diffuse reverberant loading developed by Shorter and Langley [24]. The most notable application of this relationship has been the development of hybrid junctions [25] between FEM and SEA subsystems. However, there are other important applications of the reciprocity relationship besides the development of hybrid junctions. For example, the reciprocity relationship also provides a mechanism for applying a diffuse field loading to a FE model. Shorter and Langley [24] concluded that the cross-spectral matrix of the force is proportional to the imaginary part of the direct field dynamic stiffness matrix. The input to the model is expressed mathematically as a cross-spectral force matrix. Shorter and Langley [24,27] expressed the cross-spectral force matrix ($\mathbf{S}_{ff}$) as

$$\mathbf{S}_{ff} = p^2_{\mathrm{DAF,RMS}}\frac{8\pi c}{\rho\omega^3}\mathrm{Im}\{\mathbf{D}_a\} \tag{2}$$

where $p_{\mathrm{DAF,RMS}}$ is the RMS sound pressure of the diffuse acoustic field (DAF), c is the speed of sound, ρ is the density of the fluid, and ω is the angular frequency. $\mathbf{D}_a$ is the direct field dynamic stiffness matrix of the loaded boundary. The cross-spectral force matrix ($\mathbf{S}_{ff}$) describes the reverberant loading on the FEM model. Shorter and Langley [24] demonstrated that the method was correct for radiators in an infinite rigid baffle. Ref. [24] describes the development of Eq. (2) as well as detailing how the direct field dynamic stiffness matrix at the boundary is calculated.

The radiation impedance at the termination of the plenum and duct system can be calculated in a number of ways. Probably the most straightforward approach is to compute it using either the boundary or finite element method or the Rayleigh integral method. In the current study, the radiation impedance is computed using a wavelet approach developed by Langley [26] in which Jinc functions are selected as the wavelet basis. The approach as documented in Ref. [26] is selected because of computational speed. Accordingly, the dynamic stiffness matrix for a radiating boundary into a free space ($\mathbf{D}_{rad}$) can be found by multiplying the radiation impedance by $1/i\omega$.

The plenum is modeled deterministically whereas the source and termination boundary conditions are treated in a statistical sense. Ref. [25] describes the process for determining the transmitted power. The ensemble average of the acoustic pressure response $\langle\mathbf{S}_{qq}\rangle$ of the FEM model is given by

$$\langle\mathbf{S}_{qq}\rangle = \mathbf{D}_{tot}^{-1}\mathbf{S}_{ff}\mathbf{D}_{tot}^{-\mathbf{H}} \tag{3}$$

where $\mathbf{D}_{tot}$ is the summation of the dynamic stiffness for the FEM model and the direct field dynamic stiffness at the inlet ($\mathbf{D}_a$),