Statistics and Probability Letters 110 (2016) 25–33

Contents lists available at ScienceDirect

Statistics and Probability Letters

journal homepage: www.elsevier.com/locate/stapro

Reliability study of a coherent system with single general standby component

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ARTICLE INFO

Article history:
Received 29 September 2015
Received in revised form 25 November 2015
Accepted 25 November 2015
Available online 4 December 2015

Keywords:
General standby system
System signature

ABSTRACT

The properties of a coherent system with a single general standby component is investigated. Here three different switch over viz. perfect switching, imperfect switching and random warm up period of the standby component are considered with some numerical examples.

1. Introduction

Standby allocation is one of the widely used techniques to improve the reliability of a system. Standby components are mostly of three types—cold standby, warm standby and hot standby. Cold standby means that the redundant component is inactive and has zero failure rate while in standby, and it starts to function at the time when the system/component fails. Hot standby describes the scenario where the redundant component and the corresponding system undergoes the same operational environment. In case of warm standby, the redundant component undergoes two operational environments, namely, usual environment (the environment in which the system is running) and milder environment (where the redundant component has less failure rate than that in the usual environment). Initially the warm standby component functions in milder environment and then it switches over to the usual environment at the time of system/component failure. Papageorgiou and Kokolakis (2010) derived the reliability function of a two-component parallel system with $(n-2)$ warm standby components, where two units start their operations simultaneously and any one of them is replaced instantaneously upon its failure by one of the $(n-2)$ warm standbys. Cha et al. (2008) introduced a general standby model for a single-component system with a single standby component, and derived system performance measures. The cold and the hot standby models are derived as special cases of the general standby model. Li et al. (2013, 2009) investigated some general standby systems and derived some stochastic comparison results on the lifetimes of the systems. Hazra and Nanda (2014) discussed some standby models with one and two general standby components, and compared some different series and parallel systems corresponding to the models with respect to the usual stochastic and the stochastic precedence orders. Eryilmaz (2014) derived reliability function of a coherent system equipped with a cold standby component such that the coherent system may fail at the time of the first component failure. Recently Franko et al. (2015) generalized this case by considering that the coherent system may fail at the time of sth component failure so that the standby component may

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http://dx.doi.org/10.1016/j.spl.2015.11.023
0167-7152/

be put into operation actively at the time of sth component failure. In case of warm standby redundancy, Eryilmaz (2013) investigated the reliability properties of k-out-of-n system equipped with a single warm standby component.

In this paper, we investigate the reliability properties of a coherent system equipped with a general standby component. It is to be mentioned here that the general standby studied in this paper has three different states. To be more specific, the standby component starts to work in cold state, it is switched over to warm state (from cold state) after a specified time u (≥ 0) and put into operation in active state in usual environment at the time of sth component failure which might cause the system to fail. Switching from cold state to warm state of the standby component takes place after a specified time u (> 0) only if it is known a priori that sth component failure does not occur before time u (> 0); otherwise, u must be zero, i.e., standby component starts to work in warm state from the beginning. We also consider the perfectness of the switching from one state to another state of the standby component.

The rest of the paper is organized as follows. Section 2 discusses briefly general standby model and representation of survival function of a coherent system using system signature. In Section 3, we obtain reliability of a coherent system equipped with a single general standby component in different switch over cases, namely perfect switching case, imperfect switching case and the case with random warm-up period. Numerical example is presented in this section. Finally the paper is concluded in Section 4.

For two random variables X and Y, $X =^{st} Y$ means that X and Y have the same distribution.

2. General standby model

The concepts of accelerated life tests and that of virtual age (see also Kijima, 1989, Finkelstein, 2007) have been used by Cha et al. (2008) for modeling general standby system in case of a single active component system. Let X be the lifetime of an active component with cumulative distribution function (c.d.f.) $F(\cdot)$ and let Y be the lifetime of a standby component in the usual environment with c.d.f. $G(\cdot)$. Further, let X and Y be independent. Write $\bar{F}(\cdot) = 1 - F(\cdot)$ and $\bar{G}(\cdot) = 1 - G(\cdot)$. For the standby unit in warm state, the component operates in an environment which is milder than the usual level of environment. Thus, the lifetime of the standby component in warm state will have the c.d.f. $G(\gamma(\cdot))$ where $\gamma(\cdot)$ is a non-decreasing function satisfying $\gamma(t) \leq t$, for all $t \geq 0$ with $\gamma(0) = 0$. Now, suppose that the standby unit has worked during $(0, t]$ without failure in a warm state, and is activated under usual environment at time t. Then the virtual age $\omega(t)$ of the standby component is non-decreasing satisfying $\omega(t) \leq t$, for all $t \geq 0$ and $\omega(0) = 0$. Let Y^* denote the remaining lifetime of the standby component after the failure of the active unit at time $X = x$. Then (cf. Cha et al., 2008)

$$P\{Y^* > t, I = 1 \mid X = x\} = \frac{\bar{G}(\omega(x) + t)}{\bar{G}(\omega(x))}\bar{G}(\gamma(x))$$

and the survival function of the standby system is

$$R(t) = \bar{F}(t) + \int_0^t \frac{\bar{G}(\omega(x) + t - x)}{\bar{G}(\omega(x))}\bar{G}(\gamma(x))dF(x), \tag{2.1}$$

where $\{I = 1\}$ indicates that the standby component survives at least up to the failure time of the active component. The cold and the hot standby models can be derived as special cases by substituting $\gamma(t) = \omega(t) = 0$ and $\gamma(t) = \omega(t) = t$, respectively.

In the above discussion, the standby component starts to work in warm state at the beginning. However, for a general standby component, it can be assumed that the main (active) component starts to work in active state, and the standby component starts to work in cold state, and is switched over to warm state after a pre-specified time u up to which the active component certainly does not fail. In this case, if the standby unit operates during $(u, x]$ without failure in warm state, then obviously the virtual age at time x would be $\omega(x - u)$. Now to survive the system up to time t, the following cases may arise. The active component survives up to time t, or fails in $(u, t]$ and the standby component survives for the remaining time. Clearly, if $t \leq u$, then the reliability of the system is $\bar{F}(t)$; otherwise (cf. Yun and Cha, 2010),

$$R(t) = \bar{F}(t) + \int_u^t \frac{\bar{G}(\omega(x - u) + t - x)}{\bar{G}(\omega(x - u))}\bar{G}(\gamma(x - u))dF(x). \tag{2.2}$$

3. Main results

Let T denote the lifetime of a binary coherent system without a standby and let T^{gs} denote the lifetime of the same system with a general standby component whose lifetime is Y. Suppose F is the common absolutely continuous c.d.f. of $X_1, X_2, \ldots, X_n$ having probability density function (p.d.f.) f, and G is the absolutely continuous c.d.f. of Y having p.d.f. g, where X_i denotes the lifetime of the ith component, $i = 1, 2, \ldots, n$. Here, Y and $X_1, X_2, \ldots, X_n$ are independent. Eryilmaz (2013) studied the reliability properties of the k-out-of-n system with a single warm standby component. Franko et al. (2015) studied coherent systems equipped with a cold standby component which may be put into operation at the time of the sth component failure, $s = k_\phi, k_\phi + 1, \ldots, z_\phi + 1$, where k_ϕ is the minimum number of failed components that causes the system failure whereas z_ϕ is the maximum number of failed components with which the system can still operate. In this paper, we consider coherent system equipped with a general standby component which may start to work in cold state, and is switched