Statistics and Probability Letters 110 (2016) 119–127

Contents lists available at ScienceDirect

Statistics and Probability Letters

journal homepage: www.elsevier.com/locate/stapro

Large deviations for some non-standard telegraph processes

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ARTICLE INFO

Article history:
Received 15 September 2015
Received in revised form 8 December 2015
Accepted 8 December 2015
Available online 18 December 2015

MSC:
60F10
60J27

Keywords:
Bernoulli trials
Continuous time Markov chains
Damping effect
Wave governed random motion

ABSTRACT

We prove large deviation principles for three non-standard telegraph processes. The first one is a damped model with velocity driven by Bernoulli trials studied in Crimaldi et al. (2013), and we obtain the same rate function obtained in De Gregorio and Macci (2014) for another damped telegraph process. The other telegraph processes are non-damped models and we assume suitable hypotheses: in a case the holding times have a general super-exponential distribution, in another case the change-of-direction number process satisfies a large deviation principle.

1. Introduction

The stochastic processes are often used to describe random motions. The standard telegraph process describes the motion of a particle on the real line, which moves alternatively forward and backward with finite speed. Furthermore, if one supposes that the changes of direction are governed by a homogeneous Poisson process, there is a connection with the theory of the differential equations because its density law satisfies a hyperbolic partial differential equation; in fact in some references the telegraph process is called wave governed random motion. Among the references in the literature we recall Orsingher (1990) and, since we have in mind the case with drift, Beghin et al. (2001).

Several modifications of the standard telegraph process have been studied: here we recall the damped model in Di Crescenzo and Martinucci (2010), the inhomogeneous model in Iacus (2001), the models with random drift in Orsingher and Ratanov (2002) and random velocities in Stadje and Zacks (2004), the models with jumps in Di Crescenzo et al. (2013) and Ratanov (2013), and the multidimensional models in Orsingher and De Gregorio (2007) called random flights. It is interesting to observe that applications of telegraph processes emerge in different fields. Indeed, in physics the propagation of a damped wave along a wire is described by the telegraph equation. Several recent references deal with application in finance and here we recall the monograph of Kolesnik and Ratanov (2013).

Large deviations give an asymptotic computation of small probabilities on exponential scale. Some large deviation results for telegraph processes appear in Macci (2009) and De Gregorio and Macci (2012); the first reference refers to the more general results for Markov additive processes (see e.g. Ney and Nummelin, 1987a,b,c). The aim of this paper is to present large deviation results for the following three non-standard telegraph processes.

Model 1. A particular damped model studied in Crimaldi et al. (2013). They considered a class of *damped telegraph processes with velocity driven by random trials* which depend on a parameter $A \geq 0$; here we consider the case $A = 0$ studied in Section 4 of that reference.

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http://dx.doi.org/10.1016/j.spl.2015.12.016

Models 2–3. In both cases $\{S(t) : t \geq 0\}$ starts at the origin, i.e. $S(0) = 0$, and moves with a suitable two-valued integrated telegraph signal; moreover $V(0)$ is the initial velocity and $\{N(t) : t \geq 0\}$ is the *change-of-direction number process*. A rigorous definition is the following:

$$S(t) := \int_0^t V(s)ds \quad (\text{for all } t \geq 0) \tag{1}$$

where, for some $c_1, c_2 > 0$, the velocity process $\{V(t) : t \geq 0\}$ is defined by

$$V(t) := \frac{c_1 - c_2}{2} + \frac{c_1 + c_2}{2} \left\{1_{\{V(0)=c_1\}} - 1_{\{V(0)=-c_2\}}\right\} (-1)^{N(t)}. \tag{2}$$

Moreover we assume that $V(0)$ and $\{N(t) : t \geq 0\}$ are independent, and $P(V(0) \in \{-c_2, c_1\}) = 1$.

- **Model 2**. We assume that

$$N(t) := \sum_{n\geq 1} 1_{\{\tau_1+\cdots+\tau_n\leq t\}} \tag{3}$$

  for some suitable positive super-exponential random variables $\{\tau_n : n \geq 1\}$; see the beginning of Section 3 for the hypotheses on $\{\tau_n : n \geq 1\}$.
- **Model 3**. We assume that $\{N(t)/t : t > 0\}$ satisfies the large deviation principle with a good rate function I_N and some hypotheses.

The large deviation principle for Model 1 (see Section 2) is governed by the rate function obtained in De Gregorio and Macci (2014) for a slight generalization $\{D(t) : t \geq 0\}$ of the model in Di Crescenzo and Martinucci (2010); we remark that the damping effect of the model in this paper (and in the ones in the references cited above) is due to the switching rates that grow linearly (in fact the rates of the conditional exponential distribution functions in (4) increase linearly with n).

The proof of the large deviation principle for Model 1 is based on some asymptotic estimates for the one-dimensional marginal distributions of the process. We consider a standard procedure already used in other papers: see e.g. Duffy and Sapozhnikov (2008) and De Gregorio and Macci (2014). We cannot consider the same approach for Models 2–3 because in general we do not have explicit expressions for the one-dimensional marginal distributions. However the large deviation principles for Models 2–3 can be easily proved by considering other tools, and their proofs are shorter than the one for the large deviation principle for Model 1. The author thinks that it is not possible to get a shorter proof of the large deviation principle for Model 1 by adapting the proofs considered for the other models.

The large deviation principle for Model 2 (see Section 3) is given in Proposition 3.1, and it is an easy consequence of contraction principle (see e.g. Theorem 4.2.1 in Dembo and Zeitouni (1998)) and the Example just after Corollary 5.3 in Duffy et al. (2011). We remark that, if $\{\tau_n : n \geq 1\}$ were exponentially distributed (as happens for the standard telegraph process), the proof of our result does not work; however the conclusion of Proposition 3.1 is still valid.

The large deviation principle for Model 3 (see Section 4) is given in Proposition 4.1 and it is an easy consequence of Theorem 2.3 in Chaganty (1997). Here the large deviation principle for the conditional distributions of the telegraph process given the number of changes of directions (see Proposition 2.2 in De Gregorio and Macci (2012)) plays a crucial role. Actually, since Theorem 2.3 in Chaganty (1997) deals with sequences (i.e. families of random variables with discrete time parameter), we also need to show the exponential equivalence between two processes (see e.g. Definition 4.2.10 and Theorem 4.2.13 in Dembo and Zeitouni (1998)).

We conclude with some preliminaries on large deviations (see e.g. Dembo and Zeitouni, 1998, pages 4–5). Given a topological space $\mathcal{W}$, we say that a family of $\mathcal{W}$-valued random variables $\{W(t) : t > 0\}$ satisfies the large deviation principle (LDP from now on) with rate function I if: the function $I : \mathcal{W} \to [0, \infty]$ is lower semi-continuous; the upper bound

$$\limsup_{t\to\infty} \frac{1}{t} \log P(W(t) \in C) \leq -\inf_{w\in C} I(w)$$

holds for all closed sets C; the lower bound

$$\liminf_{t\to\infty} \frac{1}{t} \log P(W(t) \in G) \geq -\inf_{w\in G} I(w)$$

holds for all open sets G. Moreover a rate function is said to be good if all its level sets $\{\{w \in \mathcal{W} : I(w) \leq \eta\} : \eta \geq 0\}$ are compact. In view of the proof of Proposition 4.1 we recall that these definitions can be formulated for a sequence of random variables (i.e. $\{W(n) : n \geq 1\}$, where n is a discrete index parameter), and for families of probability measures $\{\mu_t : t > 0\}$ on $\mathcal{W}$, where $\mu_t(\cdot) = P(W(t) \in \cdot)$. In this paper the rate functions uniquely vanish at some point x_0 and, roughly speaking, if x_0 does not belong to the closure of a set E (and therefore $\inf_{w\in E} I(w) > 0$), one can say that $P(W(t) \in E)$ goes to zero as $e^{-t\cdot\inf_{w\in E} I(w)}$ (as $t \to \infty$).