



Extremes of Gaussian fields with a smooth random variance



Goran Popivoda*, Siniša Stamatović

Department of Mathematics, University of Montenegro, P. O. Box 211, 81000 Podgorica, Montenegro

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ABSTRACT

In this paper we investigate the probabilities of large extremes $\mathbb{P}(\sup_{t \in T} \xi(t)\eta(t) > u)$, as $u \rightarrow \infty$, where $\xi(t)$ is a centered homogeneous Gaussian random field, $\eta(t)$ is particular smooth field independent of $\xi(t)$ and $T \subset \mathbb{R}^n$ is a closed Jordan set.

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1. Introduction and main result

Let $\xi(t)$, $t \in \mathbb{R}^n$, be a homogeneous Gaussian random field and $\eta(t)$ another random field being independent of $\xi(t)$. We will consider the product random field $\xi(t)\eta(t)$ investigating the tail probability of its supremum.

The tail behaviour of random variables has been studied in numerous papers, see e.g., [Jessen and Mikosch \(2006\)](#), [Hashorva et al. \(2015\)](#) and the references therein. The asymptotic theory for large extremes of Gaussian processes and fields is well developed see e.g., [Piterbarg \(1996, 2015\)](#) and [Azais and Wschebor \(2009\)](#). In the recent years several new results show the asymptotic behaviour of extremes of both smooth and non-smooth Gaussian random fields, see e.g., [Cheng \(2014\)](#), [Cheng and Schwartzman \(2015\)](#) and [Cheng and Xiao \(in press\)](#).

Random processes and fields of product type have been extensively investigated in the literature. Recently, [Piterbarg and Zhdanov \(2015\)](#) considered the product of two Gaussian processes. Analysis of extremes of product of random processes is an interesting topic which arises in various contexts of queueing theory [Arendarczyk and Dębicki \(2011, 2012\)](#), insurance mathematics [Dębicki et al. \(2014\)](#), time series analysis [Kulik and Soulier \(2015\)](#). Our investigation in this paper is in the line with those of [Hüsler et al. \(2011a\)](#), where is considered product of stationary Gaussian process $\xi(t)$ and the smooth processes $\eta(t)$, random parabola and more general process which is two times differentiable in the neighbourhood of its essential supremum. [Hüsler et al. \(2011b\)](#) considered the product of the locally stationary Gaussian process and the processes which are less smooth than those of [Hüsler et al. \(2011a\)](#).

In this paper we shall consider extremes of the product of a Gaussian random field and a paraboloid-type field, where the Gaussian random field has a constant variance and a covariance function of (E, α) type. We shall formulate below our assumptions on both the Gaussian field and the multiplier. In order to calculate the exact asymptotic behaviour of the product random field, we will calculate this probability under a fixed multiplier and then average the behaviour over

* Corresponding author.

E-mail addresses: goranp@ac.me (G. Popivoda), sins@ac.me (S. Stamatović).

all values of the multiplier. For the first part, when the multiplier is fixed, we will use well known asymptotic results for Gaussian fields.

Let the collection $\alpha = (\alpha_1, \dots, \alpha_k)$ of numbers be given, where $0 < \alpha_i \leq 2$, $i = 1, \dots, k$, as well as the collection $E = (e_1, \dots, e_k)$ of positive integers such that $\sum_{i=1}^k e_i = n$, and let us set $e_0 := 0$. Each pair (E, α) is called a *structure*, [Piterbarg \(1996\)](#). For any vector $\mathbf{t} = (t_1, \dots, t_n) \in \mathbf{R}^n$ its *structural modulus* is defined by

$$|\mathbf{t}|_{(E, \alpha)} = \sum_{i=1}^k \left(\sum_{j=E(i-1)+1}^{E(i)} t_j^2 \right)^{\frac{\alpha_i}{2}},$$

where $E(i) = \sum_{j=0}^i e_j$, $i = 1, \dots, k$. We denote by $\alpha(i)$ the number from the collection α that corresponds to i th coordinate of the vector \mathbf{t} , $i = 1, \dots, n$.

Let $T \subset \mathbf{R}^n$ be a closed Jordan set such that $\mathbf{0} \in T$. In this paper we assume that $\xi(\mathbf{t})$, $\mathbf{t} \in T$, is a homogeneous Gaussian random field with the expectation of zero and covariance function $r(\mathbf{t})$ that satisfies

$$r(\mathbf{t}) = 1 - |\mathbf{t}|_{(E, \alpha)} + o(|\mathbf{t}|_{(E, \alpha)}),$$

as $\mathbf{t} \rightarrow \mathbf{0}$, for some structure (E, α) , $E = (e_1, \dots, e_k)$, $\alpha = (\alpha_1, \dots, \alpha_k)$.

Now, let us introduce paraboloid-type field

$$\eta(\mathbf{t}) := \lambda - \frac{1}{2} \zeta \|\mathbf{t}\|^2,$$

where λ and ζ are random variables independent of $\xi(\cdot)$ ($\|\cdot\|$ denotes the Euclidean norm). We will assume that $\lambda \geq \varepsilon$ for some $\varepsilon > 0$ and ζ is positive almost surely. With the notation

$$\sigma(G) := \text{ess sup}(G)$$

for any random element G , we further assume that $\sigma(\lambda)$, $\sigma(\zeta)$, $\sigma\left(\frac{\zeta}{\lambda}\right)$ are finite.

We want to establish asymptotic exact result for

$$\mathbb{P}\left(\sup_{\mathbf{t} \in T} \xi(\mathbf{t})\eta(\mathbf{t}) > u\right), \quad \text{as } u \rightarrow \infty.$$

Note that

$$\eta(\mathbf{t}) = \lambda - \frac{1}{2} \zeta |\mathbf{t}|_{(E', \beta)},$$

where $E' = (1, \dots, 1)$ and $\beta = (2, \dots, 2)$.

The random variance of $\xi(\mathbf{t})\eta(\mathbf{t})$ conditioned on η is

$$\text{Var}(\xi(\mathbf{t})\eta(\mathbf{t}) \mid \eta(\mathbf{t})) = \eta^2(\mathbf{t}),$$

so motivated by the geometry of $\eta(\mathbf{t})$ and [Hüsler et al. \(2011a\)](#) we will call the product $\xi(\mathbf{t})\eta(\mathbf{t})$ Gaussian field with a smooth random variance.

Also note that the random covariance of $\xi \cdot \eta$ conditioned on η is

$$\text{cov}(\xi(\mathbf{t})\eta(\mathbf{t}), \xi(\mathbf{s})\eta(\mathbf{s}) \mid \eta) = \eta(\mathbf{s}) \eta(\mathbf{t}) r(\mathbf{t} - \mathbf{s}),$$

the correlation is

$$\text{Cr}(\xi(\mathbf{t})\eta(\mathbf{t}), \xi(\mathbf{s})\eta(\mathbf{s}) \mid \eta) = r(\mathbf{t} - \mathbf{s}),$$

and

$$\mathbb{E}((\xi(\mathbf{t})\eta(\mathbf{t}) - \xi(\mathbf{s})\eta(\mathbf{s}))^2 \mid \eta) = \eta^2(\mathbf{t}) - 2\eta(\mathbf{t})\eta(\mathbf{s})r(\mathbf{t} - \mathbf{s}) + \eta^2(\mathbf{s}).$$

Write

$$\Psi(x) = \frac{1}{\sqrt{2\pi x}} \exp\left\{-\frac{x^2}{2}\right\},$$

and from now on H_α denotes Pickands' constant (see [Piterbarg \(1996\)](#)).

Our main result is the next theorem.

Theorem 1. Let $\xi(\mathbf{t})$ and $\eta(\mathbf{t})$, $\mathbf{t} \in T$, be above introduced fields and let

$$\sup_{x \in T} \|x\| < \min\left(\frac{1}{\sigma(\zeta)}, \sqrt{\frac{1}{\sigma\left(\frac{\zeta}{\lambda}\right)}}\right).$$

Let the density function $f_\lambda(x)$ of the random variable λ be r times continuously differentiable in the neighbourhood of $\sigma := \sigma(\lambda)$, with $f_\lambda^{(i)}(\sigma) = 0$, $i = 0, \dots, r-1$, and $f_\lambda^{(r)}(\sigma) \neq 0$ for some $r \in \mathbf{Z}^+$.

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