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Large deviations for the stochastic present value of aggregate claims in the renewal risk model

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ABSTRACT

In insurance, if the insurer continuously invests her wealth in risk-free and risky assets, then the price process of the investment portfolio can be described as a geometric Lévy process. People always are interested in estimating the tail distribution of the stochastic present value of aggregate claims. In this paper, the large deviations for the stochastic present value of aggregate claims, when the claim size distribution is of Pareto type with finite variance, are obtained.

1. Introduction

Consider the renewal risk model in which the claim sizes, $\{X_k, k \geq 1\}$, form a sequence of independent, identically distributed (i.i.d.), and nonnegative random variables (r.v.s) with generic r.v. X and common distribution function $F(x) = P(X \leq x)$, the corresponding tail distribution function is $\overline{F}(x) = 1 - F(x)$. The inter-arrival times, $\{\theta_k, k \geq 1\}$, form another sequence of i.i.d., and nonnegative r.v.s with common distribution function $G(x)$. Let $\tau_k = \sum_{i=1}^{k} \theta_i$, $k = 1, 2, \ldots$ be the claim arrival times. Then, the number of claims up to time t is given by $N(t) = \sup\{n \geq 1, \tau_n \leq t\}$. We assume that $\lambda(t) = E(N(t)) < \infty$ for any $t \geq 0$, and $\lambda(t) \to \infty$ as $t \to \infty$. Denote $\Lambda = \{t : 0 < \lambda(t) \leq \infty\}$, $\underline{t} = \inf\{t : P(\theta \leq t) > 0\}$. It is clear that $\Lambda = [\underline{t}, \infty]$ if $P(\theta = \underline{t}) > 0$ while $\Lambda = (\underline{t}, \infty]$ if $P(\theta = \underline{t}) = 0$.

Suppose that the insurer continuously invests her wealth in risk-free and risky assets. The price process of the investment portfolio is described as a geometric Lévy process $\{e^{R_t}, t \geq 0\}$, where $\{R_t, t \geq 0\}$ is a Lévy process starting from 0. For details for general theory of Lévy process, see Sato (1999). The assumption on price process being a geometric Lévy process is widely discussed in mathematical finance, the reader is referred to the monograph of Cont and Tankov (2004) and a survey paper of Paulson (2008). In addition, we assume that $\{X_k, k \geq 1\}$, $\{N(t), t \geq 0\}$ and $\{R_t, t \geq 0\}$ are mutually independent.

Define the Laplace exponent of $\{R_t, t \geq 0\}$ by $\phi(z) = \log E(e^{-zR_1})$, $z \in (-\infty, \infty)$. If $\phi(z) < \infty$, then $E(e^{-zR_t}) = e^{t\phi(z)} < \infty$ for all $t \geq 0$. It is easy to show that $\phi(z)$ is convex in z for which $\phi(z)$ is finite. Moreover, we assume that the path of $\{R_t, t \geq 0\}$ is right continuous with left limit.

Let

$$D_n = \sum_{k=1}^{n} X_k e^{-R_{\tau_k}}, \quad n \geq 1,$$

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be the stochastic present value of the first n aggregate claims. Accordingly, the stochastic present value of aggregate claims up to time t is expressed as

$$D(t) = \sum_{k=1}^{N(t)} X_k e^{-R_{\tau_k}}, \quad t \geq 0.$$

Tang et al. (2010) investigated the tail probability of $D(t)$ in the renewal risk model. They showed that if $F \in \mathcal{R}_{-\alpha}, 0 < \alpha < \infty$ (see Section 2 for definition) and $\phi(\alpha^*) < 0$ for some $\alpha^* > \alpha$, then

$$\lim_{x\to\infty} \sup_{t\in\Lambda} \left| \frac{P(D(t) > x)}{\overline{F}(x) \int_{0-}^{t} e^{s\phi(\alpha)} d\lambda(s)} - 1 \right| = 0$$

holds. As mentioned in Tang et al. (2010), the condition $\phi(\alpha^*) < 0$ means that the impact of the insurance claims dominates that of the financial uncertainty. Noting that $\phi(0) = 0$, then the condition $\phi(\alpha^*) < 0$ implies that $\phi(z) < 0$ for all $z \in (0, \alpha^*]$.

The precise large deviation for random walk with heavy tailed increments was first studied by Heyde (1967a,b, 1968), Nagaev (1969a,b) and Nagaev (1979). Let $\{X_n, n \geq 1\}$ be i.i.d. heavy tailed random variables with a common distribution F, the precise large deviation for random walk $\{S_n = \sum_{i=1}^{n} X_i, n \geq 1\}$ is as follows. The asymptotics

$$P(S_n - E(S_n) > x) \sim n\overline{F}(x) \tag{1.1}$$

holds uniformly for some x-region T_n. The uniformity of (1.1) is understood in the following sense:

$$\lim_{n\to\infty} \sup_{x\in T_n} \left| \frac{P(S_n - E(S_n) > x)}{n\overline{F}(x)} - 1 \right| = 0. \tag{1.2}$$

In the literature, under the condition that $F \in \mathcal{R}_{-\alpha}$ with $\alpha > 2$, Nagaev (1979) proved that (1.1) holds uniformly for $x \geq \sqrt{an \ln n}$, where $a > \alpha - 2$. Further, in the case of $1 < \alpha < 2$, Cline and Hsing (1991) proved that (1.1) still holds uniformly for $x \geq a_n c_n$, where $\{a_n, n \geq 1\}$ satisfies $\lim_{n\to\infty} nP(X > a_n) = 1$ and $\{c_n, n \geq 1\}$ is any sequence satisfying $\lim_{n\to\infty} c_n = \infty$.

Tang et al. (2001) considered the large deviations of random sums $\{S(t) = \sum_{i=1}^{N(t)} X_i, t \geq 0\}$, where $\{N(t), t \geq 0\}$ is a process of non-negative integer-valued random variables, independent of $\{X_n, n \geq 1\}$. They proved that if $F \in ERV(-\alpha, -\beta)$ (see Section 2 for definition) with $1 < \alpha \leq \beta < \infty$ and that for every $\delta > 0$ and some $\varepsilon = \varepsilon(\delta) > 0$,

$$E(N(t)^{\beta+\epsilon} I_{(N(t)>(1+\delta)\lambda(t))}) = O(\lambda(t)),$$

as $t \to \infty$, then

$$\lim_{t\to\infty} \sup_{x\geq\gamma\lambda(t)} \left| \frac{P(S(t) - E(S(t)) > x)}{\lambda(t)\overline{F}(x)} - 1 \right| = 0. \tag{1.3}$$

To extend the results (1.2) and (1.3), under some mild conditions, Liu and Hu (2003) studied the probabilities of large deviations for $\{S_n, n \geq 1\}$ and $\{S(t), t \geq 0\}$ with $\{X_n, n \geq 1\}$ being independent and non-identically distributed. Tang (2006) obtained the probability of large deviation of a random walk with the increments being of a negative dependent structure.

In this paper, we will study the probability of large deviations for $\{D_n, n \geq 1\}$ and $\{D(t), t \geq 0\}$ under the assumption of $F \in \mathcal{R}_{-\alpha}, \alpha > 2$. We mention that the sequence $\{X_n e^{-R_{\tau_n}}, n \geq 1\}$ in the renewal risk model is neither independent nor identically distributed, and the conditions of our results are very simple, see Theorems 2.1 and 2.2. We also mention that Theorem 2.2 is different to the result of Tang et al. (2010) which addressed uniformity over t, Theorem 2.2 addresses uniformity on x which also reveals the relationship between x and t.

The rest of the paper is organized as follows. Section 2 presents the main results after recalling some preliminaries. Section 3 gives a series of lemmas. Section 4 proves the main results.

2. Preliminaries and main results

Throughout this paper, we assume that the distribution F of r.v.X is supported on $[0, \infty)$. We say that an r.v. X or its distribution F is heavy-tailed if and only if $E(e^{tX}) = \infty$ for all $t > 0$. Among all heavy-tailed classes, we are interested in the regularly varying class $\mathcal{R}_{-\alpha}$, $0 < \alpha < \infty$. We say that $F \in \mathcal{R}_{-\alpha}$ if

$$\lim_{x\to\infty} \frac{\overline{F}(xy)}{\overline{F}(x)} = y^{-\alpha}$$

for all $y > 0$. A slightly larger class is the class of distributions with extended regularly varying (ERV) tail. We say that $F \in ERV(-\alpha, -\beta)$ for some $0 < \alpha \leq \beta < \infty$, if

$$y^{-\beta} \leq \liminf_{x\to\infty} \frac{\overline{F}(xy)}{\overline{F}(x)} \leq \limsup_{x\to\infty} \frac{\overline{F}(xy)}{\overline{F}(x)} \leq y^{-\alpha}$$