Statistics and Probability Letters 97 (2015) 9–15

Contents lists available at ScienceDirect

Statistics and Probability Letters

journal homepage: www.elsevier.com/locate/stapro

The exponential moment tail of inhomogeneous renewal process ☆

Emilija Bernackaitė, Jonas Šiaulys *

Faculty of Mathematics and Informatics, Vilnius University, Naugarduko 24, Vilnius LT-03225, Lithuania

ARTICLE INFO

Article history:
Received 22 September 2014
Received in revised form 22 October 2014
Accepted 27 October 2014
Available online 4 November 2014

MSC:
60K05
60G50

Keywords:
Inhomogeneous renewal process
Exponential estimate
Renewal theorem

ABSTRACT

Let $\theta_1, \theta_2, \ldots$ be a sequence of nonnegative not necessarily identically distributed and independent random variables having finite means and satisfying some additional conditions. We consider the asymptotic behavior of the quantity $\mathbb{E}\left(b^{\Theta(t)}\mathbb{1}_{\Theta(t)>at}\right)$, where a and b are suitable positive constants, and $\Theta(t)$ is an inhomogeneous renewal process generated by the sequence $\theta_1, \theta_2, \ldots$. We also present a few corollaries concerning elementary renewal theorems for the above process.

1. Introduction

Klüpellberg and Mikosch (1997) considered the precise large deviation problem for the random sum

$$\sum_{i=1}^{\Theta(t)} X_i, \quad t \geqslant 0, \tag{1.1}$$

where $X_1, X_2, \ldots$ are independent identically distributed (i.i.d.) random variables with extended regularly varying distribution function (d.f.), and $\Theta(t)$ is a counting process, independent of $\{X_1, X_2, \ldots\}$, such that $\mathbb{E}\Theta(t) \xrightarrow[t\to\infty]{} \infty$ and the following two properties are satisfied:

$$(\mathcal{A}1): \frac{\Theta(t)}{\mathbb{E}\Theta(t)} \xrightarrow[t\to\infty]{\mathbb{P}} 1,$$

$$(\mathcal{A}2): \sum_{k>(1+\delta)\,\mathbb{E}\Theta(t)} \mathbb{P}(\Theta(t) \geqslant k)(1+\varepsilon)^k \xrightarrow[t\to\infty]{} 0 \quad \text{for any } \delta > 0 \text{ and some small } \varepsilon > 0.$$

It is not difficult to find examples of counting processes satisfying condition $(\mathcal{A}1)$. For instance, this condition holds for every Poisson process with unboundedly increasing accumulated intensity function and for every renewal process generated by a random variable (r.v.) θ with finite expectation $\mathbb{E}\theta$. Meanwhile, assumption $(\mathcal{A}2)$ is quite complex to verify.

☆ The authors are supported by a grant (No. MIP-13079) from the Research Council of Lithuania.

* Corresponding author.
E-mail addresses: emilija.bernackaite@mif.stud.vu.lt (E. Bernackaitė), jonas.siaulys@mif.vu.lt (J. Šiaulys).

http://dx.doi.org/10.1016/j.spl.2014.10.018
0167-7152/

Nevertheless, Klüpellberg and Mikosch (1997) (see Lemma 2.1) and Yang et al. (2013) (see Lemma 1) proved that this assumption is satisfied for a Poisson process with unboundedly increasing function $\mathbb{E}\Theta(t)$.

Tang et al. (2001) also considered the precise large deviation problem for sum (1.1). Instead of assumptions ($\mathcal{A}1$) and ($\mathcal{A}2$), they supposed that the counting process $\Theta(t)$ satisfies the following assumption:

$$(\mathcal{A}3):\quad \sum_{k>(1+\delta)\,\mathbb{E}\Theta(t)} k^{\beta}\,\mathbb{P}(\Theta(t)=k)=O(\mathbb{E}\Theta(t))\quad \text{for any } \delta>0 \text{ and some small } \varepsilon>0,$$

where $\beta>1$ is a certain number related to the regularity of d.f. $\mathbb{P}(X\leqslant x)$.

If $\mathbb{E}\Theta(t)\to\infty$ as $t\to\infty$, then assumption ($\mathcal{A}3$) follows from ($\mathcal{A}2$). So the results of Tang et al. (2001) generalize the results of Klüpellberg and Mikosch (1997) since (Tang et al., 2001) showed that assumption ($\mathcal{A}3$) implies assumption ($\mathcal{A}1$) (see Lemma 3.3) and showed that each renewal process satisfies condition ($\mathcal{A}3$) in the case where it is generated by a r.v. having a finite expectation (see Lemma 3.5).

Leipus and Šiaulys (2009) considered the asymptotic behavior of finite-time ruin probability in the renewal risk model

$$x+ct-\sum_{i=1}^{\Theta(t)} Z_i,\quad t\geqslant 0.$$

Here $x\geqslant 0$, $c>0$, $Z_1,Z_2,\ldots$ are independent identically distributed random variables with strongly subexponential d.f., and $\Theta(t)$ is a renewal process, that is,

$$\Theta(t)=\sup\{n\geqslant 1,\theta_1+\theta_2+\cdots+\theta_n\leqslant t\},\tag{1.2}$$

where $\theta_1,\theta_2,\ldots$ are independent copies of a nonnegative r.v. θ nondegenerate at zero. The authors of this paper supposed that the renewal process $\Theta(t)$ also satisfies condition ($\mathcal{A}2$) because assumption ($\mathcal{A}3$) is not sufficient to obtain the desired asymptotic formulas in the case of strongly subexponential claims $Z_1,Z_2,\ldots$. Continuing their studies on the asymptotic behavior of ruin probability, Kočetova et al. (2009) obtained that each renewal process fulfils condition ($\mathcal{A}2$) in the case where the process generator θ has a finite expectation. Namely, the following assertion was proved.

Theorem A. *Let the renewal process $\Theta(t)$ be defined in (1.2) with a sequence $\theta,\theta_1,\theta_2,\ldots$ of independent identically distributed r.v.s. If $\mathbb{E}\theta=1/\lambda\in(0,\infty)$, then for every real number $a>\lambda$, there exists $b>1$ such that*

$$\lim_{t\to\infty}\sum_{k>at}\mathbb{P}(\Theta(t)\geqslant k)\,b^k=0.\tag{1.3}$$

Chen and Yuen (2012) and Lu (2011) used this assertion considering the large deviation problem, whereas Chen et al. (2010), Bi and Zhang (2013), and Wang et al. (2012) obtained analogous assertions when the generating random variables $\theta_1,\theta_2,\ldots$ are identically distributed but dependent in some sense.

In this paper, we extend the assertion of Theorem A. We obtain that the exponential moment tail of an inhomogeneous renewal process has a similar property. We say that a process $\Theta(t)$ defined in (1.2) is an *inhomogeneous renewal process* if the generating r.v.s $\theta_1,\theta_2,\ldots$ are not necessarily identically distributed. If the r.v.s $\theta_1,\theta_2,\ldots$ are i.i.d., then we call $\Theta(t)$ a *homogeneous renewal process* or simply a *renewal process* as above. Our main results are formulated in the next section. In three theorems, we present generalizations of Theorem A, whereas in three corollaries, we show that there exist inhomogeneous renewal processes satisfying both assumptions ($\mathcal{A}1$) and ($\mathcal{A}2$).

In Theorems 2.1 and 2.2 and in Corollary 2.1, we consider an inhomogeneous renewal process generated by LEND r.v.s. In Theorem 2.2, r.v.s can be dependent in any way, whereas in Corollaries 2.2 and 2.3, we consider an inhomogeneous renewal process generated by independent r.v.s.

R.v.s $\xi_1,\xi_2,\ldots$ are said to be upper extended negatively dependent (UEND) if there exists a dominating constant α_ξ such that

$$\mathbb{P}\left(\bigcap_{k=1}^{n}\{\xi_k>x_k\}\right)\leqslant\alpha_\xi\prod_{k=1}^{n}\mathbb{P}(\xi_k>x_k)$$

for all n and all $x_1,x_2,\ldots,x_n$.

Similarly, r.v.s $\xi_1,\xi_2,\ldots$ are said to be lower extended negatively dependent (LEND) if there exists a dominating constant β_ξ such that

$$\mathbb{P}\left(\bigcap_{k=1}^{n}\{\xi_k\leqslant x_k\}\right)\leqslant\beta_\xi\prod_{k=1}^{n}\mathbb{P}(\xi_k\leqslant x_k)$$

for all n and all $x_1,x_2,\ldots,x_n$.

One can find related concepts of negative dependence and useful properties of negatively dependent r.v.s, for instance, in Tang (2006), Liu (2009), and Chen et al. (2010).

By showing assertions of our corollaries we prove a so-called elementary renewal theorem for an inhomogeneous renewal process. Of course, this elementary renewal theorem can be derived from well-known classical results (see, for instance, Kawata, 1956, Hatori, 1959, 1960 and Smith, 1964). However, we show that this theorem can be also obtained using an analog of Theorem A.