STAPRO: 7122 Model 3Gsc pp. 1–7 (col. fig: NIL)

Statistics and Probability Letters xx (xxxx) xxx–xxx

Contents lists available at ScienceDirect

Statistics and Probability Letters

journal homepage: www.elsevier.com/locate/stapro

Stochastic comparisons of parallel systems with exponentiated Weibull components

Longxiang Fang[a,b,*], Xinsheng Zhang[b]

[a] *Department of Statistics, Anhui Normal University, Wuhu 241000, PR China*

[b] *Department of Statistics, Fudan University, Shanghai 200433, PR China*

ARTICLE INFO

Article history:
Received 25 March 2014
Received in revised form 21 October 2014
Accepted 27 October 2014
Available online xxxx

MSC:
60E15
62N05
62G30
62D05

Keywords:
Exponentiated Weibull distribution
Parallel systems
Usual stochastic order
Dispersive order
Likelihood ratio order
Majorization

ABSTRACT

The exponentiated Weibull (EW) distribution is the first generalization of the two-parameter Weibull distribution to accommodate nonmonotone hazard rates, including the bathtub shaped hazard rate. In this paper, we discuss stochastic comparisons of parallel systems with exponentiated Weibull components in terms of the usual stochastic order, dispersive order and the likelihood ratio order. We give some sufficient conditions for stochastic comparisons between lifetimes of parallel systems.

1. Introduction

In probability and statistics theory, the two-parameter Weibull distribution has been most commonly used in reliability and life testing, extreme value theory and many other areas due to its distribution with several desirable properties and nice physical interpretations. Recently, Prabhakar Murthy et al. (2004) have studied two-parameter Weibull models in detail. But often, the hazard rate of lifetime data has increasing (decreasing) shape, the bathtub shape or upside-down bathtub shape. Since two-parameter Weibull distribution has monotone hazard rate, it is not a proper statistical model to accommodate nonmonotone hazard rates. To this end, this has lead to the need to seek generalizations of the two-parameter Weibull distribution. The exponentiated Weibull (EW) family of distributions was constructed by Mudholkar and Srivastava (1993) by using exponentiation on the two-parameter Weibull family of distributions. That is, the EW distribution with three parameters is a generalization of the commonly known two-parameter Weibull distribution. And the EW distribution is quite adequate for modeling non-monotone failure rates, including the bathtub shaped hazard rate, which are quite common in reliability and biological studies. It has been shown in the literature that the EW distribution has significantly better fit than traditional models based on the exponential, gamma, Weibull and log-normal distributions in many cases.

* Corresponding author at: Department of Statistics, Anhui Normal University, Wuhu 241000, PR China.
E-mail address: lxfang@fudan.edu.cn (L. Fang).

http://dx.doi.org/10.1016/j.spl.2014.10.017

Please cite this article in press as: Fang, L., Zhang, X., Stochastic comparisons of parallel systems with exponentiated Weibull components. Statistics and Probability Letters (2014), http://dx.doi.org/10.1016/j.spl.2014.10.017

A random variable X is said to have the exponentiated Weibull (EW) distribution if its cumulative distribution function is given by

$$F(x;\alpha,\beta,\lambda)=\left(1-e^{-(\lambda x)^{\beta}}\right)^{\alpha},\quad x>0,\alpha>0,\beta>0,\lambda>0, \tag{1.1}$$

Here, α and β are shape parameters and λ is a scale parameter, respectively. $EW(\beta,\alpha,\lambda)$ would be used to denote a EW distribution. From (1.1), we know that the EW distribution includes many distributions as special cases. If $\alpha=1$, it reduces to the standard two-parameter Weibull distribution. If $\beta=1$, it reduces to the generalized exponential(EE) distribution due to Gupta and Kundu (1999). If $\alpha=1$ and $\beta=1$, it becomes the one-parameter exponential distribution. The particular case for $\beta=2$ is the Burr type X distribution studied by Sartawi and Abu-Salih (1991), Aludaat et al. (2008) and among others. The particular case for $\beta=2$ and $\alpha=1$ is the Rayleigh distribution. The detailed review of some basic properties of the EW distribution is contained in Nadarajah and Kotz (2006) and Nadarajah et al. (2013).

We give two applications of the EW distribution as the following:

(i) Construct a parallel system consisting of n independent and identically distributed components with common two-parameter Weibull distribution $EW(\beta,1,\lambda)$. Then, the lifetime of the system has a EW distribution $EW(\beta,n,\lambda)$. In other words, the family of EW distributions is closed under maxima, that is, if $X_1,\ldots,X_n$ are independent random variables with $EW(\beta,\alpha_i,\lambda)$, $i=1,\ldots,n$, then $\max(X_1,\ldots,X_n)$ also has a EW distribution $EW(\beta,\sum_{i=1}^{n}\alpha_i,\lambda)$. Therefore, the EW distribution is closed under parallel structure with respect to the shape parameter α while the Weibull distribution which is closed under series structure.

(ii) Let $X_1,\ldots,X_n$ be independent Weibull random variables such that $X_1,\ldots,X_p$ with $EW(\beta_1,1,\lambda_1)$ and $X_{p+1},\ldots,X_n$ with $EW(\beta_2,1,\lambda_2)$. This model is called as multiple-outlier Weibull model. Note that $\max(X_1,\ldots,X_n)=\max(Y_1,Y_2)$, where Y_1 and Y_2 are independent random variables such that $Y_1\sim EW(\beta_1,p,\lambda_1)$ and $Y_2\sim EW(\beta_2,n-p,\lambda_2)$. Thus, a parallel system with the multiple-outlier Weibull components can be considered as a parallel system with two heterogeneous EW components.

Let $X_1,\ldots,X_n$ be n random variables and let $X_{i:n}$ denote their ith order statistic. In reliability theory, k-out-of-n systems consisting of n components work as long as at least k components are working. The survival function of a k-out-of-n system is the same as that of the $(n-k+1)$th order statistic $X_{n-k+1:n}$ of a set of n random variables. In particular, the parallel systems are 1-out-of-n systems. Thus, the study of parallel systems is equivalent to the study of the largest order statistics. For k-out-of-n systems or $(n-k+1)$-out-of-n systems, order statistics have been studied in the literature, for example, see Gurlerand Bairamov (2009), Gurler and Capar (2011) and Gurler (2012). Generally, the results of stochastic comparisons of order statistics can be seen in Balakrishnan et al. (2014), Khaledi and Kochar (2000, 2002), Khaledi et al. (2011), Kochar and Xu (2009a,b), Kochar (2012), Misra et al. (2011) and so on.

In this paper, we will investigate stochastic comparisons of parallel systems with exponentiated Weibull components. Some recent results on the EW distribution can be found in the literature of Gurler and Capar (2011), Mudholkar and Srivastava (1993), Mudholkar et al. (1995) and Shen et al. (2009). To continue our discussion, we need definitions of some stochastic orders and the concept of majorization which is given in Section 2. In this paper, we compare the lifetime of two parallel systems with independent EW components. Firstly, let $X_1,\ldots,X_n$ be independent random variables with $X_i\sim EW(\beta_i,\alpha,\lambda)$, $i=1,\ldots,n$. Let $X_1^*,\ldots,X_n^*$ be independent random variables with $X_i^*\sim EW(\beta_i^*,\alpha,\lambda)$, $i=1,\ldots,n$. We will show that $X_{n:n}\ge_{st}X_{n:n}^*$ under the condition of $(\beta_1,\ldots,\beta_n)\succeq_m(\beta_1^*,\ldots,\beta_n^*)$. Secondly, let $X_1,\ldots,X_n$ and $X_1^*,\ldots,X_n^*$ be two independent EW random variables with common shape parameter β and varying other two parameters. We consider stochastic comparisons in terms of the usual stochastic order, dispersive order and the likelihood ratio order.

2. Preliminaries

We first introduce some notations. Suppose that the random variables X and Y have distribution functions $F(x)$ and $G(x)$, density functions $f(x)$ and $g(x)$, the survival functions $\overline{F}(x)=1-F(x)$ and $\overline{G}(x)=1-G(x)$, respectively. And we define the quantile functions as $F^{-1}(p)=\inf\{x:F(x)\ge p\}$ and $G^{-1}(p)=\inf\{x:G(x)\ge p\}$, for all real values $p\in(0,1)$. In this paper, all the random variables are assumed to be nonnegative and having support $(0,+\infty)$. Let $R=(-\infty,+\infty)$, $R_+=(0,+\infty)$, $R^n=\{(x_1,\ldots,x_n):x_i\in R \text{ for all } i\}$.

Definition 2.1. Let X and Y be two nonnegative random variables having support $(0,+\infty)$,

(i) X is said to be less dispersive than Y if

$$F^{-1}(\beta)-F^{-1}(\alpha)\le G^{-1}(\beta)-G^{-1}(\alpha),$$

for $0<\alpha\le\beta<1$, in symbols, $Y\ge_{disp}X$.

(ii) X is said to be smaller than Y in the likelihood ratio order if $\frac{g(x)}{f(x)}$ is increasing in $x\in R_+$, in symbols, $Y\ge_{lr}X$.

(iii) X is said to be smaller than Y in the usual stochastic order if $\overline{F}(x)\ge\overline{G}(x)$ for all x; in symbols, $Y\ge_{st}X$.

For a comprehensive discussion on various stochastic orders, one may refer to Shaked and Shanthikumar (1994).

Majorization is a very interesting topic in statistics, which is a pre-ordering on vectors by sorting all components in increasing order.

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