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Large deviations for one-dimensional random walks on discrete point processes

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ABSTRACT

Berger and Rosenthal introduced a random walk model on discrete point processes, that is, a simple random walk defined on a random subset of $\mathbb{Z}^d$. In this article, we study both the quenched and the annealed large deviations for the one-dimensional case. Their rate functions are linked via a variational formula, analogous to the classical one-dimensional random walk in random environment.

1. Introduction and main results

A random walk on discrete point processes is a random walk on a random subset of $\mathbb{Z}^d$. Given a random subset of $\mathbb{Z}^d$, a simple random walk is defined on this random subset so that it moves to the nearest neighbor in each coordinate with equal probability. The model was introduced by Rosenthal (2010) and Berger and Rosenthal (in press). They studied law of large numbers, transience and recurrence and quenched central limit theorem for any dimension $d \geq 1$. More recently, Kubota (2013) proved a quenched invariance principle for the model. In this paper, we study the large deviations for this model in the one-dimensional case.

1.1. Random walks on discrete point processes

Let us introduce precisely the model for one-dimensional case. Let $\Omega = \{0, 1\}^{\mathbb{Z}}$ and $\omega = (\omega_x)_{x\in\mathbb{Z}} \in \Omega$ is a random environment. For any $x \in \mathbb{Z}$, $\omega_x = 1$ if x is a point in the environment and $\omega_x = 0$ otherwise. Let $\mathcal{B}$ be the Borel σ-algebra with respect to the product topology on Ω and we define the shift operator $\theta_x : \Omega \to \Omega$ as $(\theta_x\omega)_y = \omega_{x+y}$ for any $x, y \in \mathbb{Z}$.

Let Q be the probability measure on Ω so that

- Q is stationary and ergodic with respect to the shifts θ_x, $x \in \mathbb{Z}$.
- $Q(\omega : \omega_x = 0 \text{ for all } x \in \mathbb{Z}) < 1$.

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Let $\Omega_0 = \{\omega \in \Omega : \omega_0 = 1\}$. Since we assumed $Q(\omega : \omega_x = 0 \text{ for all } x \in \mathbb{Z}) < 1$, $Q(\Omega_0) > 0$ and define P on Ω_0 as the conditional probability measure of Q, i.e., for any Borel set $A \in \mathcal{B}$,

$$P(A) = Q(A|\Omega_0) = \frac{Q(A \cap \Omega_0)}{Q(\Omega_0)}. \tag{1.1}$$

Let $N_x(\omega)$ be the set of nearest neighbors of point x in ω, i.e.,

$$N_x(\omega) = \min\{y > x : \omega(y) = 1\} \cup \max\{y < x : \omega(y) = 1\}. \tag{1.2}$$

Given an environment ω, we define a random walk on ω as the Markov chain with the transition probabilities given by

$$P^\omega(X_{n+1} = y | X_n = x) = \begin{cases} \frac{1}{2} & \text{if } y \in N_x(\omega) \\ 0 & \text{otherwise,} \end{cases} \tag{1.3}$$

and the random walk starts at 0, i.e., $P^\omega(X_0 = 0) = 1$. P^ω is called the quenched probability measure in the literature. The semi-direct product $\mathbb{P} = P \times P^\omega$ is referred to as the annealed probability measure.

In terms of limit theorems, the law of large numbers is the same under both the quenched and annealed cases, while large deviations can exhibit different limiting behavior depending on whether it is quenched or annealed.

One-dimensional random walk on discrete point processes has the following representation and like most one-dimensional random walk in random media models, which is much more tractable than higher-dimensional cases.

Let $Z_n = Z_n(\omega)$ denote the position of the nth point. More precisely, $Z_0 := 0$, $Z_n := \inf\{x > Z_{n-1} : \omega_x = 1\}$ and $Z_{-n} := \sup\{x < Z_{-(n-1)} : \omega_x = 1\}$, for any $n \geq 1$.

Let Y_n be a simple random walk on $\mathbb{Z}$, such that $Y_0 = 0$ and

$$\begin{cases} Y_{n+1} = Y_n + 1 & \text{with probability } \frac{1}{2} \\ Y_{n+1} = Y_n - 1 & \text{with probability } \frac{1}{2}. \end{cases} \tag{1.4}$$

Then, we can write the random walk X_n and Z_{Y_n} have the same distribution. Using this representation, Berger and Rosenthal (in press) proved that for P-a.e. ω, for any $x \in \mathbb{R}$,

$$\lim_{n\to\infty} P^\omega\left(\frac{X_n}{\sqrt{n}} \leq x\right) = \Phi\left(\frac{x}{E[Z_1]}\right), \tag{1.5}$$

where $\Phi(x) := \int_{-\infty}^x \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du$ is the cumulative function for standard Gaussian. Indeed, if one follows the proofs in Berger and Rosenthal (in press), one can easily check that the annealed central limit theorem also holds, i.e., for any $x \in \mathbb{R}$,

$$\lim_{n\to\infty} \mathbb{P}\left(\frac{X_n}{\sqrt{n}} \leq x\right) = \Phi\left(\frac{x}{E[Z_1]}\right). \tag{1.6}$$

Moreover, it is easy to see that the law of the iterated logarithms holds for both quenched and annealed case, i.e., for $\mathbb{P}$ a.s., or equivalently, for a.e. ω and P^ω a.s.

$$\limsup_{n\to\infty} \frac{X_n}{\sqrt{2n \log\log n}} = \limsup_{n\to\infty} \frac{Z_{Y_n}}{Y_n} \frac{Y_n}{\sqrt{2n \log\log n}} = E[Z_1]. \tag{1.7}$$

For random walks on discrete point processes in higher dimensions, Berger and Rosenthal (in press) used the fact that the random walk is reversible and applied Kipnis–Varadhan theory, see Kipnis and Varadhan (1986) to obtain the quenched central limit theorem.

1.2. Large deviations for random walks on discrete point processes

Before we proceed, recall that a sequence $(P_n)_{n\in\mathbb{N}}$ of probability measures on a topological space X satisfies the large deviation principle with rate function $I : X \to \mathbb{R}$ if I is non-negative, lower semicontinuous and for any measurable set A, we have

$$-\inf_{x\in A^o} I(x) \leq \liminf_{n\to\infty} \frac{1}{n} \log P_n(A) \leq \limsup_{n\to\infty} \frac{1}{n} \log P_n(A) \leq -\inf_{x\in\overline{A}} I(x). \tag{1.8}$$

Here, A^o is the interior of A and $\overline{A}$ is its closure. A rate function $I(x)$ is said to be good if the level sets $\{x : I(x) \leq \alpha\}$ are compact for any α, see e.g. p. 4 of Dembo and Zeitouni (1998).

We refer to Dembo and Zeitouni (1998) or Varadhan (1984) for general background of large deviations and the applications.