Statistics and Probability Letters 97 (2015) 125–131

Contents lists available at ScienceDirect

Statistics and Probability Letters

journal homepage: www.elsevier.com/locate/stapro

On residual inaccuracy of order statistics

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ARTICLE INFO

Article history:
Received 28 May 2014
Received in revised form 28 October 2014
Accepted 9 November 2014
Available online 18 November 2014

Keywords:
Order statistics
Residual entropy
Hazard rate
Survival function

ABSTRACT

We propose the measure of residual inaccuracy of order statistics and prove a characterization result for it. Further we characterize some specific lifetime distributions using residual inaccuracy of the first order statistics. We also discuss some properties of the proposed measure.

1. Introduction

Let X and Y be two non-negative random variables with pdf respectively f and g. Shannon (1948) measure of uncertainty associated with X and Kullback (1959) measure of discrimination of X about Y are given by respectively

$$H(X) = H(f) = -\int_0^\infty f(x)\log f(x)dx, \tag{1}$$

and

$$H(f|g) = \int_0^\infty f(x)\log\left(\frac{f(x)}{g(x)}\right)dx. \tag{2}$$

Adding (1) and (2), we get

$$H(f) + H(f|g) = -\int_0^\infty f(x)\log g(x)dx, \tag{3}$$

which is Kerridge (1961) measure of inaccuracy associated with random variables X and Y. If we consider F as the actual distribution function then G can be interpreted as reference distribution function.

In survival analysis and life testing, the current age of the system under consideration is also taken into account. Thus, for calculating the remaining uncertainty of a system which has survived up to time t, the measures defined in (1)–(3) are not suitable. Ebrahimi (1996) considered a random variable $X_t = (X - t)|X > t,\ t \geq 0$ and defined uncertainty and discrimination of such a system, given by

$$H(f;t) = -\int_t^\infty \frac{f(x)}{\bar{F}(t)}\log\left(\frac{f(x)}{\bar{F}(t)}\right)dx, \tag{4}$$

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http://dx.doi.org/10.1016/j.spl.2014.11.010

and

$$H(f|g;t) = \int_t^\infty \frac{f(x)}{\bar{F}(t)} \log\left(\frac{f(x) \setminus \bar{F}(t)}{g(x) \setminus \bar{G}(t)}\right) dx \tag{5}$$

respectively, where $\bar{F}(t) = 1 - F(t)$.

Clearly when $t = 0$, then (4) and (5) reduce respectively to (1) and (2).

Taneja et al. (2009) defined dynamic measure of inaccuracy associated with two residual lifetime distributions F and G corresponding to the Kerridge measure of inaccuracy given by

$$H(f,g;t) = -\int_t^\infty \frac{f(x)}{\bar{F}(t)} \log\left(\frac{g(x)}{\bar{G}(t)}\right) dx. \tag{6}$$

Clearly for $t = 0$, it reduces to (3).

In this communication we propose the measure of residual inaccuracy of order statistics. By the term order statistics we mean if $X_1, X_2, \ldots, X_n$ are n independent and identically distributed observations from a distribution F, where F is differentiable with a density f which is positive in an interval and zero elsewhere, then the order statistics of a sample is defined by the arrangement of $X_1, X_2, \ldots, X_n$ from the smallest to the largest denoted as $X_{1:n}, X_{2:n}, \ldots, X_{n:n}$. These statistics have been used in a wide range of problems like detection of outliers, characterizations of probability distributions, testing strength of materials etc., for details refer to Arnold et al. (1992) and David and Nagaraja (2003). In reliability theory order statistics are used for statistical modeling, as the ith order statistics in a sample of size n corresponds to life length of a $(n-i+1)$-out-of-n system. The pdf of the ith order statistics $X_{i:n}$ is given by

$$f_{i:n}(x) = \frac{1}{B(i, n-i+1)} F(x)^{i-1}(1-F(x))^{n-i} f(x), \tag{7}$$

where $B(i, n-i+1) = \frac{\Gamma(i)\Gamma(n-i+1)}{\Gamma(n+1)}$ is the beta function with parameters i and $(n-i+1)$, for details refer to Arnold et al. (1992).

Several authors have worked on information theoretic aspects of order statistics, for details refer to Ebrahimi et al. (2004) and Zarezadeh and Asadi (2010). Recently Thapliyal and Taneja (2013) have introduced the concept of inaccuracy using order statistics. They have proposed the measure of inaccuracy between the ith order statistics and the parent random variable and proved a characterization result for the same. In this paper we extend the concept of inaccuracy of ordered random variables to dynamical system. The organization of this paper is as follows: In Section 2 we propose the measure of residual inaccuracy for the ith order statistics and explore some properties of it. Section 3 focuses on characterization results based on residual inaccuracy of order statistics. Some concluding remarks are mentioned in Section 4.

2. Measure of residual inaccuracy for $X_{i:n}$

Ebrahimi et al. (2004) studied some information theoretic measures based on order statistics using probability integral transformation and defined Shannon entropy and Kullback relative information measures, which are given by respectively

$$H_n(X_{i:n}) = H_n(f_{i:n}) = -\int_0^\infty f_{i:n}(x) \log f_{i:n}(x) dx = H_n(W_{i:n}) - E_{g_i}[\log f(F^{-1}(W_i))], \tag{8}$$

and

$$H_n(f_{i:n}; f) = \int_0^\infty f_{i:n}(x) \log\left(\frac{f_{i:n}(x)}{f(x)}\right) dx = -H_n(W_{i:n}), \tag{9}$$

where $W_{i:n}$ is the ith order statistics of uniformly distributed random variables $U_1, U_2, \ldots, U_n$ and

$$g_i(w) = \frac{1}{B(i, n-i+1)} w^{i-1}(1-w)^{n-i}, \quad 0 \le w \le 1,$$

is density function of $W_{i:n}$.

Thapliyal and Taneja (2013) defined the inaccuracy between the ith order statistics and the parent random variable as

$$I_n(f_{i:n}, f) = -\int_0^\infty f_{i:n}(x) \log(f(x)) dx. \tag{10}$$

Analogous to (10), we propose

$$I_n(f_{i:n}, f; t) = -\int_t^\infty \frac{f_{i:n}(x)}{\bar{F}_{i:n}(t)} \log\left(\frac{f(x)}{\bar{F}(t)}\right) dx, \tag{11}$$